Defining eccentricity for gravitational wave astronomy

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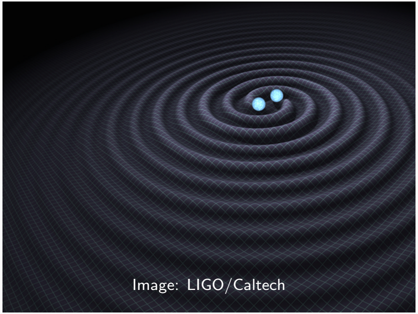
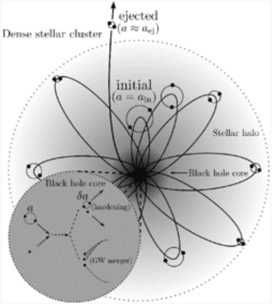
So far GW data analysis use quasicircular waveforms



- About 90 [Abbott et al., 2021a] CBCs have been detected → includes BBH [Abbott et al., 2016], BHNS [Abbott et al., 2021b], BNS [Abbott et al., 2017] systems.
- Analysed using quasicircular waveform models → eccentricity = 0
- Binaries formed in galactic fields → isolated evolution → lose eccentricity as it inspirals [Peters and Mathews, 1963, Peters, 1964] → circularization

GW from eccentric binary require eccentric waveform models

- Dynamical formation → highly eccentric binary [Mapelli, 2020]
  - globular cluster via direct capture [Rodriguez et al., 2019, Rodriguez et al., 2018, Rodriguez et al., 2016, Samsing et al., 2014, Samsing et al., 2018]
  - galactic center [Antonini and Rasio, 2016]
  - Field triples via Kozai-Lidov oscillation [Naoz, 2016, Antonini et al., 2017]
- Require eccentric model for detection and analysis of these signals.



J. Samsing (2017)

Existing eccentric waveform models

- Post-Newtonian
  - EccentricTD [Tanay et al., 2016]
  - EccentricFD [Huerta et al., 2014]
- Effective One Body
  - SEOBNRE [Cao and Han, 2017, Liu et al., 2020] SEOBNREHM [Liu et al., 2022]
  - SEOBNRv4EHM [Ramos-Buades et al., 2022]
  - TEOBResumS [Nagar et al., 2021, Chiaramello and Nagar, 2020, Nagar et al., 2018]
- Numerical Relativity
  - SpEC [SXS Collaboration,]
  - RIT [Healy and Lousto, 2022]
- Numerical Relativity Surrogate → NRSur2dq1Ecc [Islam et al., 2021] NRSur3dq4Ecc (ongoing)

A few issues with current models

- In GR, pericenter precesses → binary orbit is no longer closed → no unique definition of eccentricity → gauge dependence [Mora and Will, 2002]
- Incompatible definitions of eccentricity → model dependence [Knee et al., 2022]
- Neglecting mean anomaly as a free parameter [Islam et al., 2021, Clarke et al., 2022]
- Lack of a standardized definition → ambiguity in PE inference.

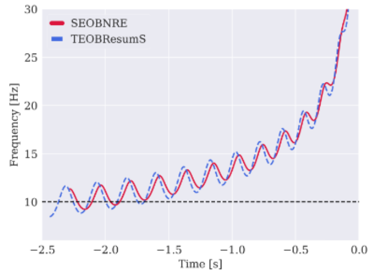


Image: A. Knee (2022)

Defining Eccentricity: required features

- Three parameters: eccentricity e and mean anomaly l at given reference frequency f_{ref}.
- Gauge independent and model independent
- Reduces to Keplerian eccentricity in Newtonian limit
- Applicable to full range of eccentricity $(0 - 1)$ for bound orbits.
- Applicable to waveforms of different origins.
- Computationally cheap.

Defining eccentricity using gravitational waveform

Define eccentricity the oscillations in the frequency or amplitude of the gravitational waveform. [Ramos-Buades et al., 2020, Islam et al., 2021, Ramos-Buades et al., 2022, Bonino et al., 2022]

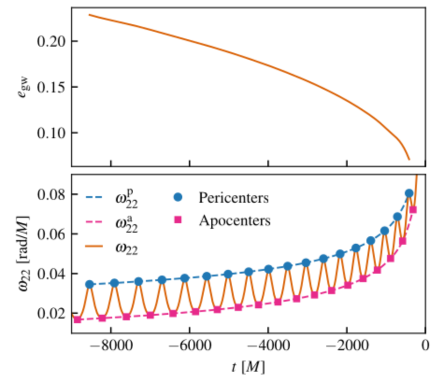


Image: Shaikh+ (2023)

$$h_+ - ih_\times = \sum_{\ell=2}^{\ell=\infty} \sum_{m=-\ell}^{m=\ell} h_{\ell m}(\lambda, t)\, {}_{-2}Y^{\ell m} \tag{1}$$

$$e_{\omega_{22}} = \frac{\sqrt{\omega_{22}^{p}(t)} - \sqrt{\omega_{22}^{a}(t)}}{\sqrt{\omega_{22}^{p}(t)} + \sqrt{\omega_{22}^{a}(t)}} \tag{2}$$

$$\omega_{22} = \frac{d\phi_{22}}{dt} \qquad h_{22} = A_{22} e^{-\phi_{22}} \tag{3}$$

$$e_{gw} = \cos(\Psi/3) - \sqrt{3}\sin(\Psi/3) \tag{4}$$

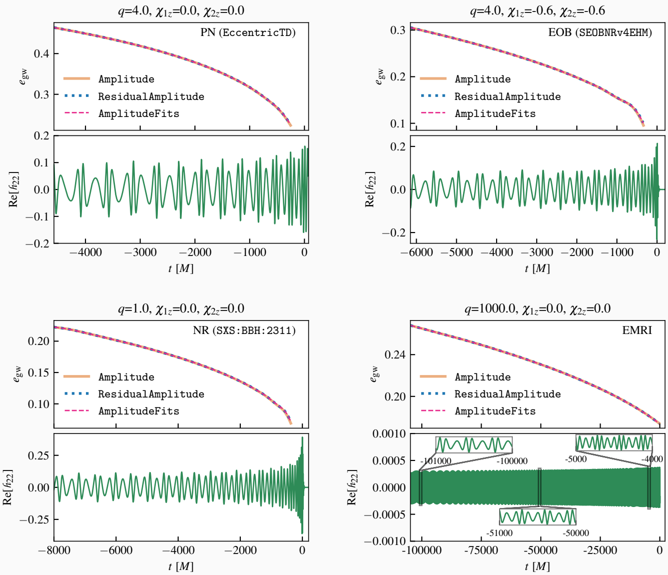
$$\Psi = \arctan\left(\frac{1 - e_{\omega_{22}}^2}{2\, e_{\omega_{22}}}\right) \tag{5}$$

Implementation package: gw_eccentricity

https://pypi.org/project/gw-eccentricity/

- Public Python package gw_eccentricity to measure eccentricity and mean anomaly from GW waveform.
- Implemented 6 different methods to compute eccentricity.
- Can measure $e_{\mathrm{gw}} \in (0-1)$ from early inspiral to close to the merger.
- Very robust $\rightarrow$ works for waveforms of different origins.
- Can be applied in post-processing step of Parameter Estimation to rule out ambiguity due to model definitions.

Application to different waveform models



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General Relativity and Quantum Cosmology

[Submitted on 22 Feb 2023]

Defining eccentricity for gravitational wave astronomy

Md Arif Shaikh, Vijay Varma, Harald P. Pfeiffer, Antoni Ramos-Buades, Maarten van de Meent

Eccentric compact binary mergers are significant scientific targets for current and future gravitational wave observatories. To detect and analyze eccentric signals, there is an increasing effort to develop waveform models, numerical relativity simulations, and parameter estimation frameworks for eccentric binaries. Unfortunately, current models and simulations adopt different internal parameterisations of eccentricity in the absence of a unique natural definition of eccentricity in general relativity, which can result in incompatible eccentricity measurements. In this paper, we present a standard definition of eccentricity and mean anomaly based solely on waveform quantities. This definition is free of gauge ambiguities, has the correct Newtonian limit, and can be applied as a postprocessing step when comparing eccentricity measurements from different models. This standardization puts all models and simulations on the same footing and enables direct comparisons between eccentricity estimates from gravitational wave observations and astrophysical predictions. We demonstrate the applicability of our definition for waveforms of different origins, including post-Newtonian theory, effective one body, extreme mass ratio inspirals, and numerical relativity simulations. We focus on binaries without spin-precession in this work, but possible generalizations to spin-precessing binaries are discussed. We make our implementation publicly available through an easy-to-use Python package, gw_eccentricity.

Comments: Python implementation available at this https URL
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; High Energy Astrophysical Phenomena (astro-ph.HE)
Cite as: arXiv:2302.11257 **[gr-qc]**
(or arXiv:2302.11257v1 **[gr-qc]** for this version)

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gw-eccentricity 1.0.0

```
pip install gw-eccentricity
```

Latest version

Released: Feb 24, 2023

Defining eccentricity for gravitational wave astronomy.

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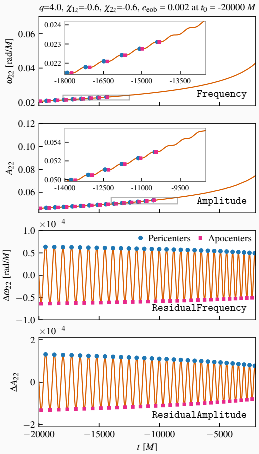
Eccentricity measurement methods: `Amplitude` and `Frequency`

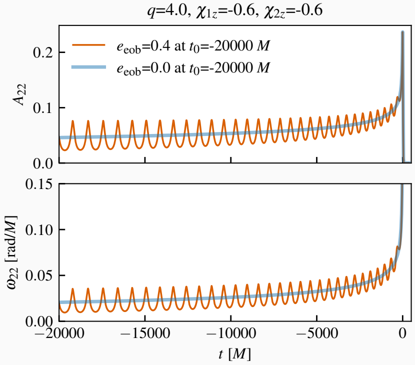
Each method is named after the data $U(t)$ it uses for finding the pericenter/apocenter.



- `Amplitude` or `Frequency` uses $U(t) = A_{22}$ or ω_{22}
- Works for only relatively large eccentricity $\gtrsim 10^{-3}$

$$e_{\rm gw} \gtrsim \frac{192}{15}\nu\left(\frac{M\omega_{22}}{2}\right)^{5/3}. \qquad (6)$$

ResidualAmplitude and ResidualFrequency



- Uses residual data
- For ResidualFrequency

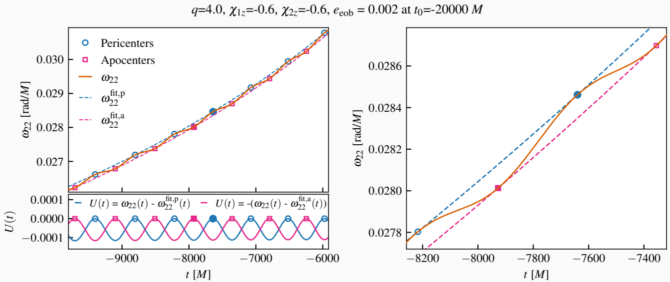
$$U(t) = \Delta\omega_{22}(t) \equiv \omega_{22}(t) - \omega_{22}^{\rm circ}(t), \tag{7}$$

and likewise for the ResidualAmplitude

$$U(t) = \Delta A_{22}(t) \equiv A_{22}(t) - A_{22}^{\rm circ}(t), \tag{8}$$

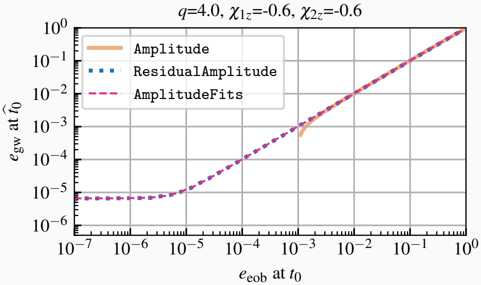
- Works for full range of $e_{\rm gw} \in (0 - 1)$

AmplitudeFits and FrequencyFits



- It uses residual data $U(t) = \omega_{22}(t) - \omega_{22}^{\text{fit,p}}(t)$, where $\omega_{22}^{\text{fit,p}}(t;\, A,\, n,\, t_{\rm merg}) = A(t_{\rm merg} - t)^n$
- Works for full range $(0 - 1)$
- Less reliable than ResidualAmplitude or ResidualFrequency.

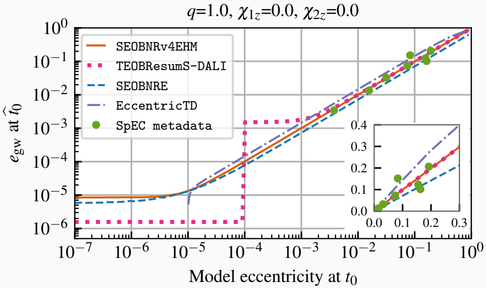
Applicable to full range of eccentricity



Shaikh+ (2023)

- Residual/Fits Can measure eccentricity $e_{gw} \approx 10^{-5}$ to $e_{gw} \approx 1.0$
- Amp/Freq fails for $e_{gw} \lesssim 10^{-3}$
- Highlights that waveform model no longer producing distinguishable waveforms below $e_{eob} \lesssim 10^{-5}$.

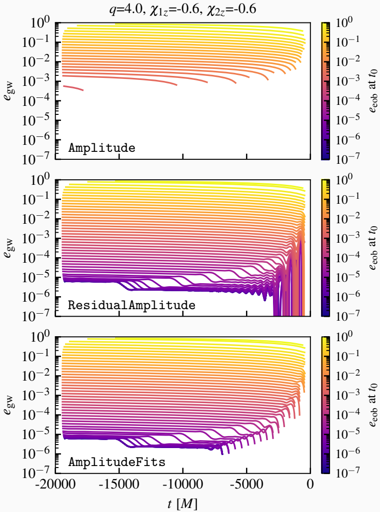
Measured eccentricity e_{gw} vs model eccentricity



Shaikh+ (2023)

- The models differs significantly at low eccentricity.
- `TEOBResumS-DALI` has a minimum eccentricity $10^{-4} \to e_{gw} > 10^{-3}$
- `EccentricTD` has a minimum eccentricity 10^{-5}
- `SEOBNRv4EHM` and `SEOBNRE` has $e_{gw} \gtrsim 10^{-5}$

Evolution of measured eccentricity $e_{\rm gw}$



- Using a set of $\approx 20,000M$ long `SEOBNRv4EHM` waveforms.
- $e_{\rm gw}$ varies smoothly with time.
- The colors represent the initial $e_{\rm eob}$ at $t_0 = -20,000M$
- `Amplitude` works for only $e_{\rm gw} \gtrsim 10^3$
- For smaller $e_{\rm eob}$, `Amplitude` stops far from the merger.
- The jumps in `ResidualAmplitude` and `AmplitudeFits` highlights issues in the waveform model.

Application in PE

Apply gw_eccentricity to measure eccentricity directly from waveforms at the sample parameters and reconstruct the posterior on eccentricity.

Summary and Remarks

Summary

- We implement a standardized definition of eccentricity and mean anomaly.
- This definition is model-independent, gauge-independent.
- Reduces to the well known Keplerian definition of eccentricity in the Newtonian limit.
- We provide public package `gw_eccentricity` with several methods to measure eccentricity.
- Our implementation is robust and applies to different waveform models.

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