Stationary black holes and stars in the Brans-Dicke theory with $\Lambda > 0$ revisited

Md Sabir Ali

School of Physical Science and Technology, LanZhou University, LanZhou, Gansu, P.R. China

Asia-Pacific School and Workshop on Gravitation and Cosmology 2023

May 15, 2023

Outline of the talk

Outline of the talk

Introduction and Motivation

- Long-range forces $\longrightarrow$ the gravitational field $g_{\mu\nu}$, and the electromagnetic fields A_μ
- It is natural to ask whether the long-range forces may be transmitted through some scalar fields
- Brans-Dicke theory shares the stage with gravitation $\longrightarrow$ based on Mach's principle
- The Brans-Dicke theory is a prototype of the gravitaional theories alternative to general relativity [1]

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right] + S^{(\mathrm{m})}$$

[1] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925-935 (1961), C. H. Brans, Phys. Rev. **125**, 2194-2201 (1962)

Outline of the talk

- The action for the matter part $S^{(\mathrm{m})} = \int d^4x\sqrt{-g}\mathcal{L}^{(\mathrm{m})}$
- The field equations for $V(\phi) = 2\Lambda$:

$$\Box\phi = \frac{T - 4\Lambda}{2\omega + 3} \qquad (\omega \neq -3/2)$$

$$R_{\mu\nu} = \frac{\Lambda(2\omega+1)}{\phi(2\omega+3)}g_{\mu\nu} + \frac{T_{\mu\nu}}{\phi} - \frac{T(\omega+1)}{\phi(2\omega+3)}g_{\mu\nu} + \frac{\omega}{\phi^2}(\nabla_\mu\phi)(\nabla_\nu\phi) + \frac{\nabla_\mu\nabla_\nu\phi}{\phi}$$

$$R = \frac{2\omega(4\Lambda - T)}{\phi(2\omega+3)} + \frac{\omega}{\phi^2}(\nabla_\mu\phi)(\nabla^\mu\phi)$$

- $\omega \to \infty$, $\Box\phi = \mathcal{O}(1/\omega) \to \phi = \langle\phi\rangle + \mathcal{O}(1/\omega) = 1/G + \mathcal{O}(1/\omega)$
- Immediately, we have[2] $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = T^{(m)}{}_{\mu\nu} + \mathcal{O}(1/\omega)$

[2] S. Weinberg, "*Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*," John Wiley and Sons, 1972

Outline of the talk

Nonexistence of black holes

To start with a generic static and spherically symmetric metric:

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

The ϕ-equation becomes

$$D_\mu(\sqrt{f}D^\mu\phi) = -\sqrt{f}\,\frac{4\Lambda - T^{(m)}}{2\omega + 3} \tag{2}$$

For $T^{(m)}_{\mu\nu} = 0$ and multiplying both side of (2) by $e^{\epsilon\phi}$, and then integrating it

$$\int_{\partial\Sigma}\sqrt{\frac{f}{h}}e^{\epsilon\phi}\partial_r\phi = \int_\Sigma e^{\epsilon\phi}\left[\epsilon D_\mu\phi D^\mu\phi - \frac{4\Lambda}{2\omega+3}\right] \tag{3}$$

Regularity of the field $\phi(r)$ and its derivative $\partial_r\phi$ **l.h.s.** of Eq. (3),
$\int_{\partial\Sigma}\sqrt{\frac{f}{h}}e^{\epsilon\phi}\partial_r\phi = 0$
$D_\mu\phi D^\mu\phi \geq 0$ and setting $\epsilon = \pm1$ for $2\omega + 3 > 0(< 0)$, $\phi = const$, $\omega^{-1} = 0$

The existence of the cosmological horizon in addition to the black hole event horizon rules out the Brans-Dicke theory, $|\omega| \gtrsim 40,000$

At large scalas ϕ becomes very strong and thereby screening the Λ as is confirmed from the first of the field equations itself.

We fix the boundary conditions only at the black hole event horizon

We take the Schwarzschild-de Sitter (SdS) geometry near the black hole event horizon at the leading order

$$ds^2|_{\mathbf{BH}} \to -(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

For $T^{(m)}_{\mu\nu} = 0$, the ϕ-equation has the generic solution[3]

$$\phi(r)|_{\mathbf{BH}} = c_2 + \frac{2}{2\omega+3}\Big[\frac{c_1}{r_H}\ln\left(1-\frac{r_H}{r}\right) + \left(1-\frac{c_1}{2r_C}\right)\ln\left(1-\frac{r}{r_C}\right) + \left(1+\frac{c_1}{2r_C}\right)\ln\left(1+\frac{r}{r_C}\right)\Big]$$

We must set $c_1 = 0$ and as $\omega \to \infty \to \phi \to 1$ to get regular $\phi(r)$

$$\phi(r)|_{\mathbf{BH}} = 1 + \frac{2}{2\omega+3}\ln\left(1-\frac{r^2}{r_C^2}\right)$$

For $\Lambda = 0$, $r_C \to \infty$, we recover the no-hair theorem
$r_C \sim 10^{26}m$, and $r_H \sim 10^{16}m$, and $\omega \gtrsim 10^4$, $\phi \to 1 + \mathcal{O}(10^{-24})$
$f(r \to r_H) = h(r \to r_H)^{-1} = \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} - \frac{\Lambda r^4}{5\omega r_C^2}\right)$
For $r_H \sim 10^{16}$, $\frac{2M}{r} \sim \mathcal{O}(1)$, $\frac{\Lambda r^2}{3} \sim \mathcal{O}(10^{-20})$, and $\frac{\Lambda r^4}{5\omega r_C^2} \sim \mathcal{O}(10^{-45})$

[3] S. Bhattacharya, K. F. Dialektopoulos, A. E. Romano and T. N. Tomaras, Phys. Rev. Lett. **115**, 181104 (2015) [arXiv:1505.02375 [gr-qc]].

$\phi \to \infty$ is unphysical and unacceptable

We can also extend to rotating de Sitter solution

$$\phi(r \to r_H) = 1 + \phi(r) + \phi(\theta)$$

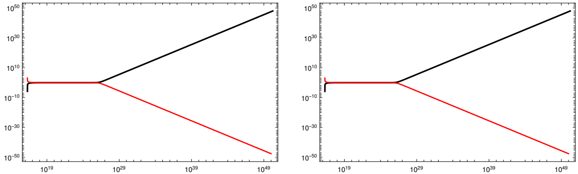

Figure: Metric function as a function of radial distance

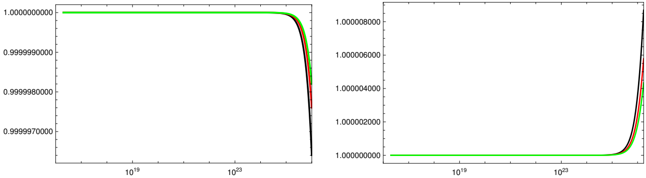

Figure: The field $\phi(r)$ as a function of radial distance

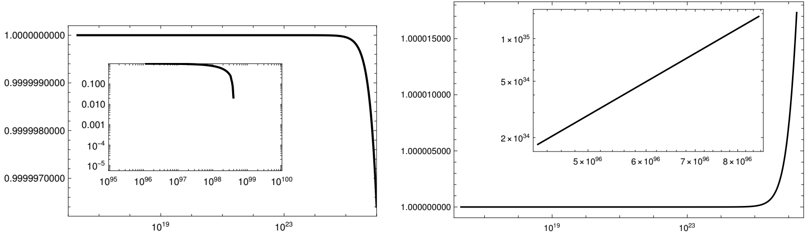

Figure: *$\phi(r)$ vs r, depicting its pathological behaviour at large radial distances.*

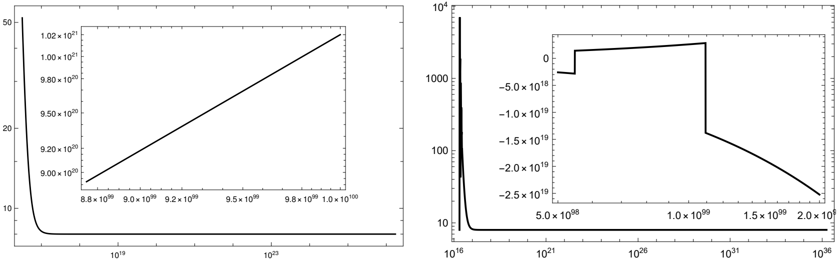

Figure: *The variation of the Ricci scalar R, vs r.*

Outline of the talk

We assume that the trace of the energy momentum-tensor constituting the star is less than or equal to zero[4]

We also assume that the centre of the star is flat, we have lesser and lesser matter fields to create gravity

Integrate Eq. (2) from $r = 0$ up to the star surface to have for $d\phi/dr$,

$$4\pi r^2 \sqrt{\frac{f}{h}} \frac{d\phi}{dr}\Big|_{r=R_0} = -\frac{1}{2\omega+3}\int_{\Sigma}^{R_0} \sqrt{f}\,(4\Lambda - T) \tag{4}$$

Next integrate (2) from $r = R_0$ up to some $r = r_0$ outside the star and use (4) into it to have

$$4\pi r^2 \sqrt{\frac{f}{h}} \frac{d\phi}{dr}\Big|_{r=r_0} = -\frac{1}{2\omega+3}\int_{\Sigma, r=0}^{R_0} \sqrt{f}\,(4\Lambda - T) - \frac{4\Lambda}{2\omega+3}\int_{\Sigma, R_0}^{r_0} \sqrt{f}$$

$d\phi/dr$ is monotonically decreasing (increasing) for $2\omega + 3 > 0$ ($2\omega + 3 < 0$).

[4] R. M. Wald, *General Relativity*, Chicago Univ. Press (USA), 1984

Outline of the talk

Conclusions

The non-existence of regular stationary black hole and star solutions in the Brans-Dicke theory for $\Lambda > 0$

It is reasonable to expect a cosmological event horizon. But one cannot *a priori* rule out possible alternative boundary conditions where the Brans-Dicke field $\phi(r)$ is strong

We would like to emphasise the stark contrast that while the standard no hair theorems only talk about the field configurations, a parameter of the theory gets constrained, not only for black holes but also for stars

We rule out the non-triviality due to a positive Λ, that was chiefly related to the exotic boundary effects.

It seems to be an interesting task to check the cosmological anisotropy dissipation/no hair theorem in the Brans-Dicke theory with $\Lambda > 0$

Thank You