Complexity Equals Anything: Singularity Probes

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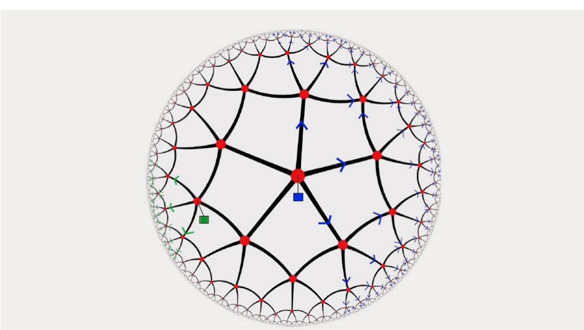
arXiv:2111.02429 arXiv:2210.09647 with A. Berlin, R. Myers, G. Sarosi, A. Seperanza

arXiv:2304.05453 with E. Jørstad, R. Myers



It From Qubit Collaboration

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

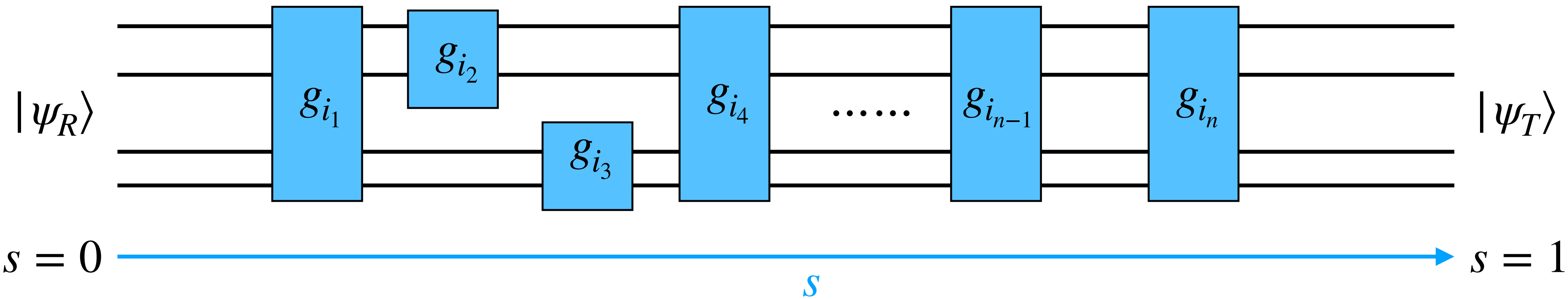
Extreme Universe Collaboration

Outline

- Motivations and Background
- Circuit Complexity and Holographic Complexity
- Complexity=Anything
- Singularity Probes

What is Circuit Complexity?

Quantum Circuit



$$|\psi_{\rm T}\rangle = U_{\rm TR}\,|\psi_{\rm R}\rangle = g_{i_n}\cdots g_{i_2}\,g_{i_1}\,|\psi_{\rm R}\rangle\,,$$

Target state   Gates   Reference state

Circuit Complexity

The minimal number(cost) of gates in quantum circuits $|\Psi_R\rangle \longrightarrow |\Psi_T\rangle \equiv U_{TR}|\Psi_R\rangle$

What is Circuit Complexity?

The minimal number of gates in quantum circuits for $|\Psi_R\rangle \longrightarrow |\Psi_T\rangle \equiv U_{TR}|\Psi_R\rangle$

Circuit Complexity: $\mathcal{C}\left(|\Psi_{\rm T}\rangle\right) = \min \mathcal{D}(U(\sigma)) = \min \int_0^1 ds\, F\left(U(s), Y^I(s)\right)$

For other approaches toward the complexity in QFT, see eg, :
1707.08582, S. Chapman, M. P. Heller, H. Marrochio and F. Pastawski;
1703.00456, 1706.07056, P. Caputa, N. Kundu, M. Miyaji, T. Takayanagi, K. Watanabe;

Why Circuit Complexity?

Free Choices (Ambiguities)?

References State?	Set of Gates?	Cost Functions?
Unentangled State?	Minimal Set?	?

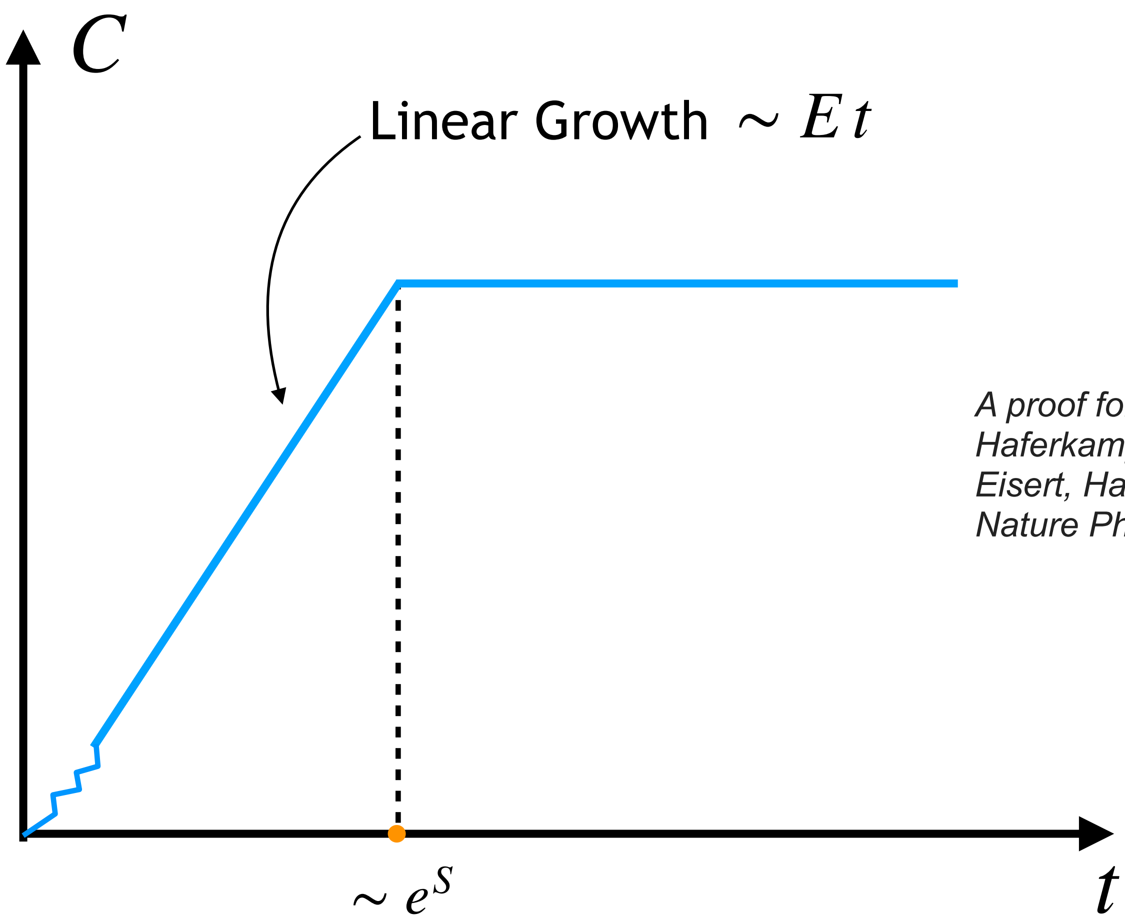
02. Circuit Complexity

Why Circuit Complexity?

Universalities: Linear growth and Switchback effect

A conjecture of the time evolution of complexity

$$|\psi_{\mathrm{T}}(t)\rangle = e^{-iHt}|\psi\rangle$$

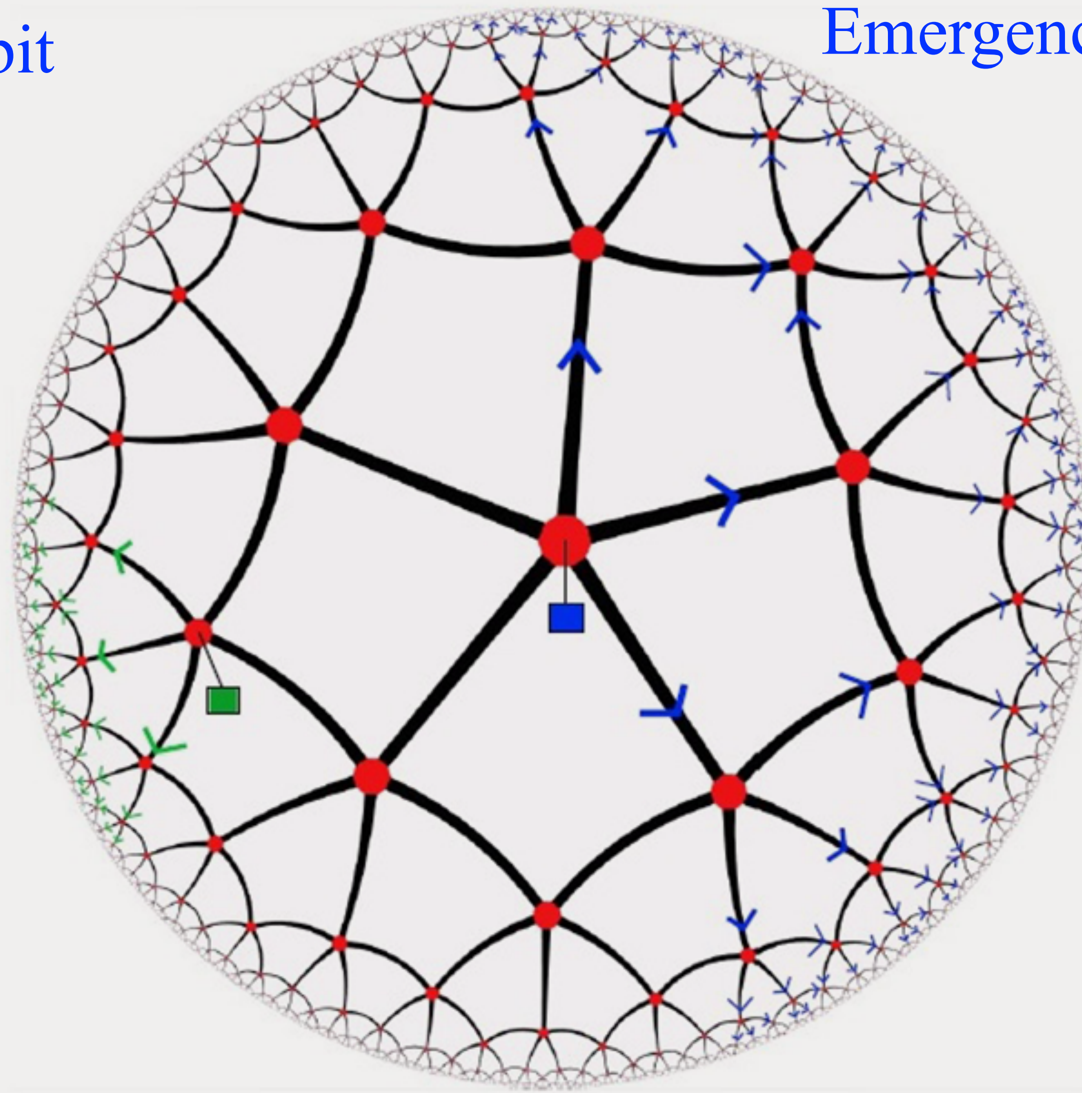


A proof for random circuits of qubits
Haferkamp, Faist, Kothakonda, Eisert, Halpern
Nature Physics, *18*(5), 528-532

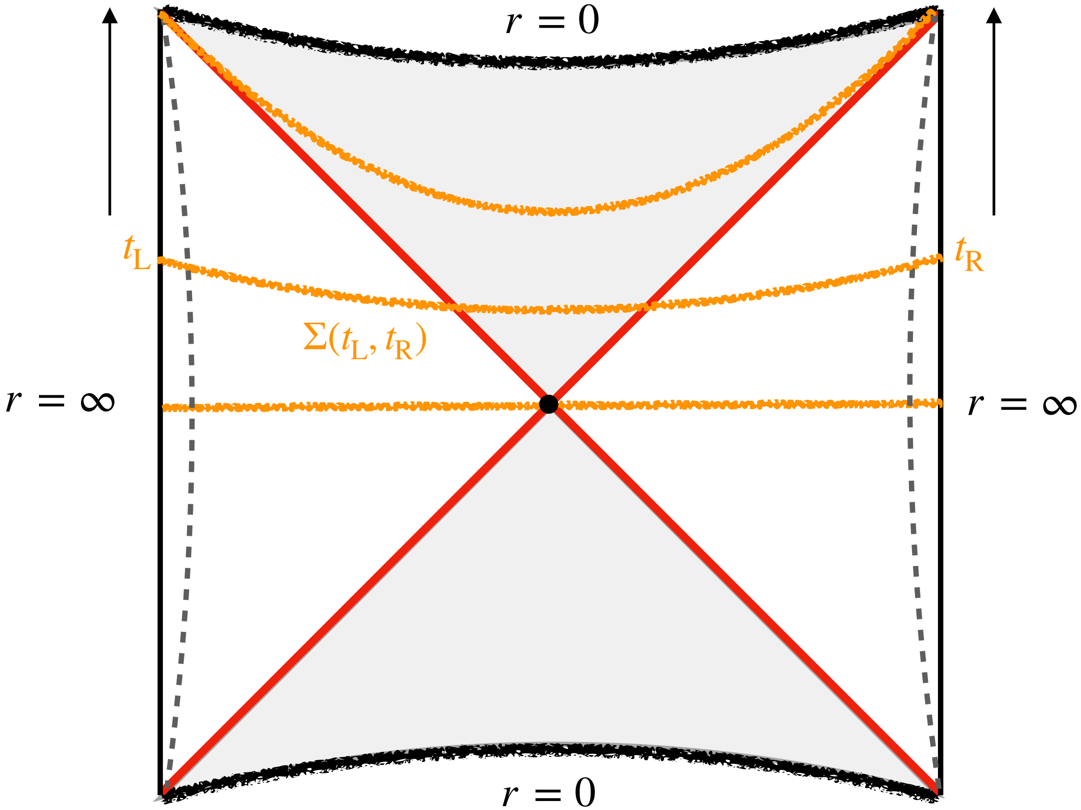
02. “Quantum Circuit”

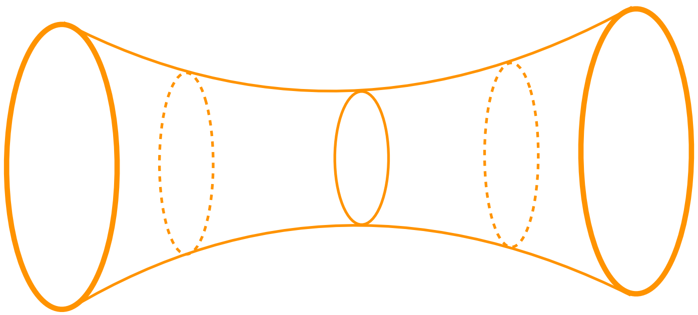
It from Qubit

Emergence of Spacetime



Universality: Linear growth of the size of wormhole





size $\sim t_L + t_R$

Geometries behind the horizon

$$\mathcal{C}_{\rm V} = \max \left[\frac{\mathcal{V}}{G_N \ell_{\rm bulk}} \right]$$

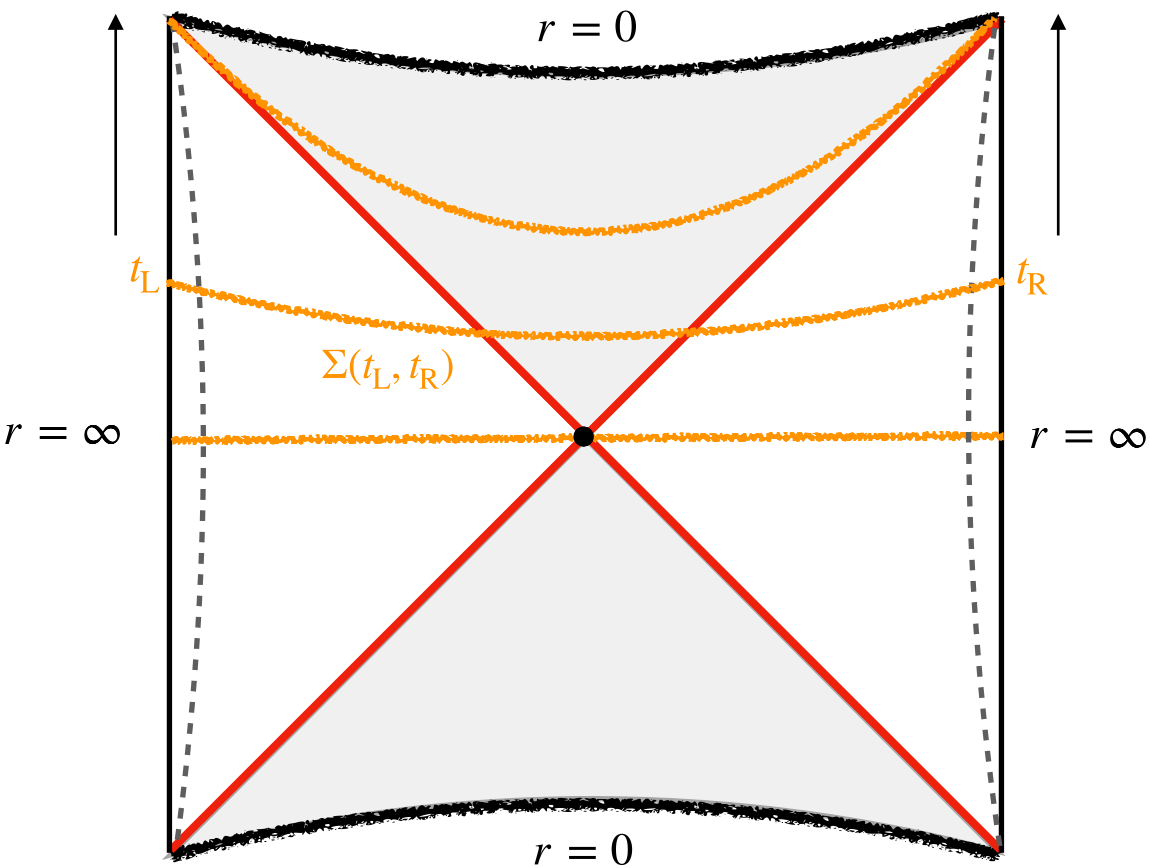
Ambiguities: Complexity Equals Anything?

arXiv:2111.02429 arXiv:2210.09647

A. Berlin, R. Myers, S-M Ruan, G. Sarosi, A. Seperanza

03. Complexity Equals Anything

$$O_{F_1,\Sigma_{F_2}}\left(\Sigma_{\mathrm{CFT}}\right) = \frac{1}{G_{\mathrm{N}} L}\int_{\Sigma_{F_2}} d^d\sigma\sqrt{h}F_1\left(g_{\mu\nu};X^\mu\right)$$



arXiv:2111.02429, A. Berlin, R. Myers, SM.Ruan, G. Sarosi, A. Seperanza

Extremal hypersurface

$$\delta_{\mathrm{x}}\left(\int_{\Sigma} d^d\sigma\sqrt{h}F_2\left(g_{\mu\nu};X^\mu\right)\right) = 0$$

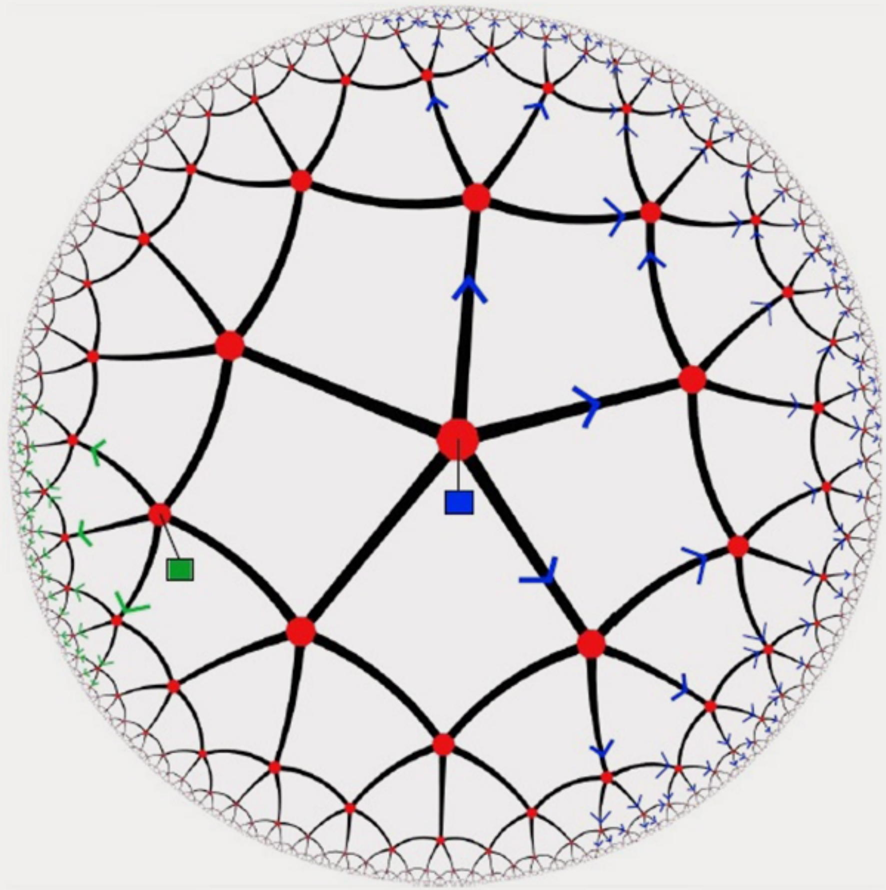
$F_1 = F_2 = 1$

$$\mathcal{C}_{\mathrm{V}} = \max\left[\frac{\mathcal{V}}{G_N \ell_{\mathrm{bulk}}}\right]$$

03. Complexity Equals Anything

$$F_1 \neq F_2 \qquad O_{F_1,\Sigma_{F_2}}(\Sigma_{\mathrm{CFT}}) = \frac{1}{G_{\mathrm{N}} L}\int_{\Sigma_{F_2}} d^d\sigma \sqrt{h} F_1(g_{\mu\nu}; X^\mu)$$

$F_2 = 1$ → Extremal Surface



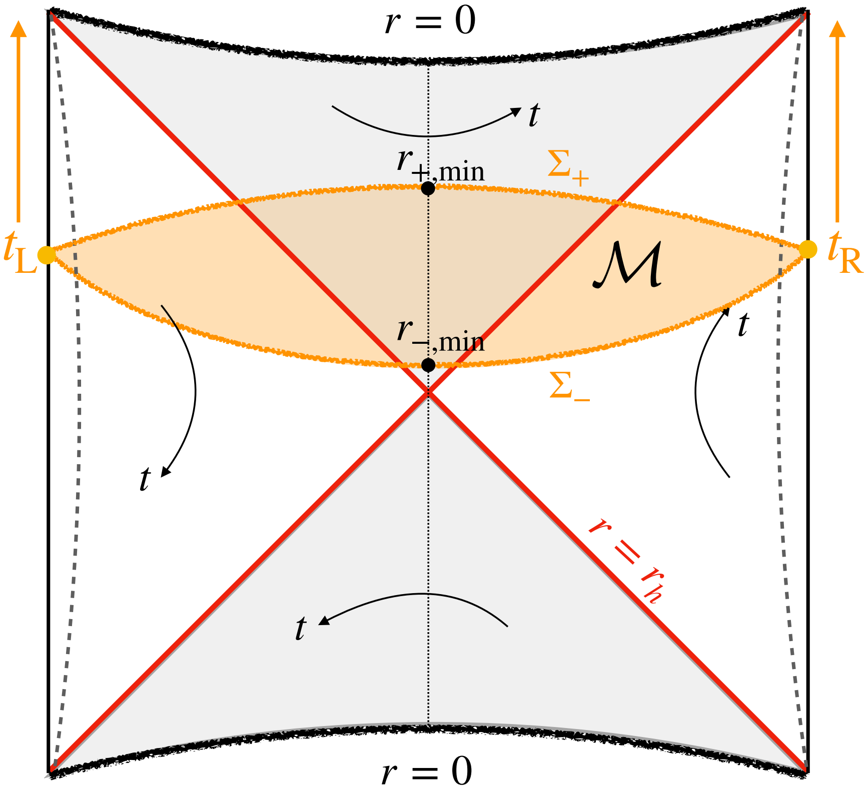
F_1 → Complexity measure

A generic codimension-zero gravitational observables:

$$O\left[G_1, F_{1,\pm}, \mathcal{M}_{G_2,F_{2,\pm}}\right](\Sigma_{\mathrm{CFT}}) = \frac{1}{G_{\mathrm{N}}L}\int_{\Sigma_+[G_2,F_{2,+}]} d^d\sigma\sqrt{h}F_{1,+}\left(g_{\mu\nu}; X^\mu_+\right)$$

$$+\frac{1}{G_N L}\int_{\Sigma_-[G_2,F_{2,-}]} d^d\sigma\sqrt{h}F_{1,-}\left(g_{\mu\nu}; X^\mu_-\right) + \frac{1}{G_N L^2}\int_{\mathcal{M}_{G_2,F_{2,\pm}}} d^{d+1}x\sqrt{g}G_1\left(g_{\mu\nu}\right)$$

Codimension-one | Codimension-zero

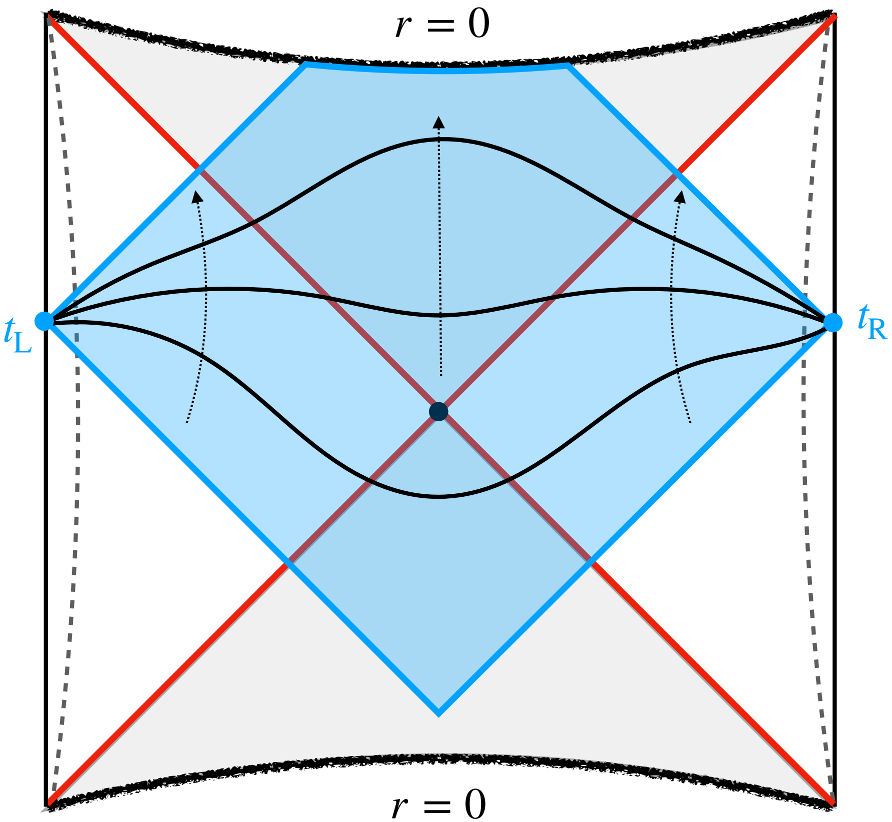


Extremal Codimension-zero Subregion

$$\delta_{\mathrm{x}_\pm}\left[W_{G_2,F_{2,\pm}}(\mathcal{M})\right]=0$$

04. Singularity Probe

$$\mathcal{C}_{\text{gen}} = \frac{1}{G_N L}\int_\Sigma F_1\left(g_{\mu\nu}, \mathcal{R}_{\mu\nu\rho\sigma}, \nabla_\mu\right)\sqrt{h}d^d y$$
$$= \frac{V_x}{G_N L}\int d\sigma \left(\frac{r}{L}\right)^{d-1}\sqrt{-f(r)\dot{v}^2 + 2\dot{v}\dot{r}}\, a(r)$$



(Maximal) extremal hypersurface

↓

Singularity

Singularity Probes:

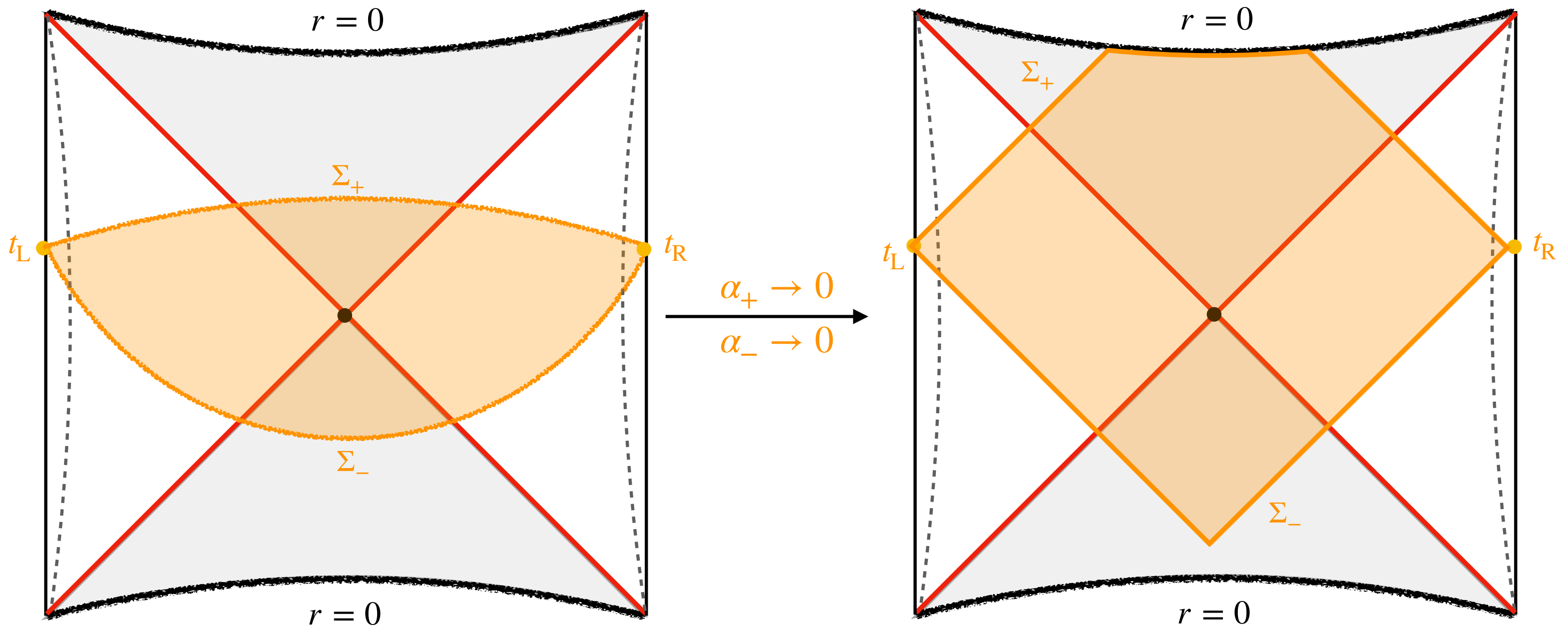
01. Asymptotic geometries near the singularity

02. Distinguish spacelike/timelike singularity

Simplest Example: $F = G = 1$
Spacetime volume between CMC slices

$$\mathcal{C}_{\text{gen}} = \frac{1}{G_{\text{N}} L} \left[\alpha_+ \int_{\Sigma_+} d^d\sigma \sqrt{h} + \alpha_- \int_{\Sigma_-} d^d\sigma \sqrt{h} + \frac{\alpha_{\text{B}}}{L} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g} \right],$$

Extremization Conditions: $K_{\Sigma_+} = \frac{\alpha_{\text{B}}}{\alpha_+ L}, \quad K_{\Sigma_-} = -\frac{\alpha_{\text{B}}}{\alpha_- L}$ constant mean curvature (CMC)



Lessons and Questions

- Infinite Gravitational Observables
- Finding extremal hypersurface —> EoMs of a classical particle
- Linear Growth & Switchback effect
- Two Independent Measures: Extremal Surface + Complexity Measure
- All as candidates for holographic complexity?
- Higher curvature corrections & Quantum corrections C_{bulk}?
- A derivation of holographic complexity?
- ……….

Thanks for your attention!