Fates of bounded orbits in backgrounds of exotic compact objects

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1. Introduction

Boson star (mimicker of black hole)

- horizonless (globally regular solitons with localized structures and finite energy)[a];
- GW190521 can be modeled as the merger of binary Proca stars [b];
- spherical Proca star can accurately give the same shadow as the Schwarzschild black hole[c].

[a] D. A. Feinblum and W. A. McKinley, Phys. Rev. **168**, 1445 (1968); D. J. Kaup, Phys. Rev. **172**, 1331 (1968); R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969); E. W. Mielke and R. Scherzer, Phys. Rev. D **24**, 2111 (1981);

[b] J. C. Bustillo, N. Sanchis-Gual, A. Torres-Forné, J. A. Font, A. Vajpeyi, R. Smith, C. Herdeiro, E. Radu, and S. H. W. Leong, Phys. Rev. Lett. **126**, 081101 (2021);

[c] C. Herdeiro, A. M. Pombo, E. Radu, P. V. P. Cunha, and N. Sanchis-Gual, JCAP **04**, 051 (2021).

boson star can be classified into three types in terms of the stability[a]:

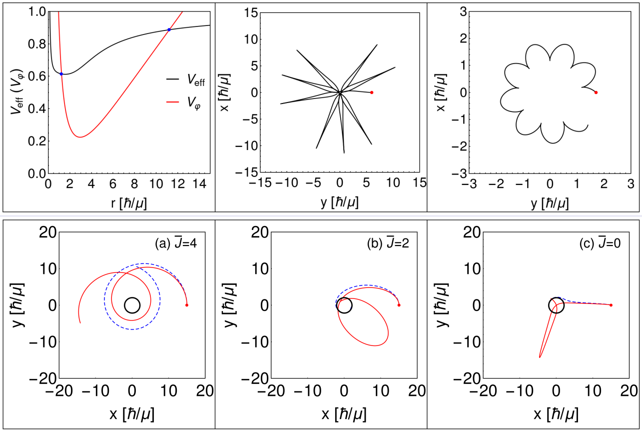
[a] N. Sanchis-Gual, C. Herdeiro and E. Radu, Class. Quant. Grav. **39**, 064001 (2022).

- an unstable boson star will collapse into a black hole;
- an unstable boson star will migrate into the stable state;
- a stable boson star will remain unchanged.

1. Introduction

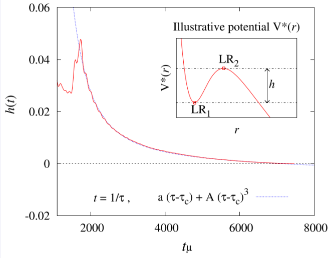
geodesics in boson stars[abc] ($j = -0.248, m = 1, \omega = 0.79$)

[a]M. Kesden, J. Gair, and M. Kamionkowski, Phys. Rev. D **71**, 044015 (2005);
[b]L. G. Collodel, B. Kleihaus, and J. Kunz, Phys. Rev. Lett. **120**, 201103 (2018);
[c]Y.-P. Zhang, Y.-B. Zeng, Y.-Q. Wang, S.-W. Wei, and Y.-X. Liu, Phys. Rev. D **105**, 044021 (2022).



boson star stability and geodesics

- a boson star could possess a pair of stable and unstable light rings, such boson star will be unstable [a];
- it will collapse into black holes or migrate into the stable state, and finally the light-rings disappear[b].



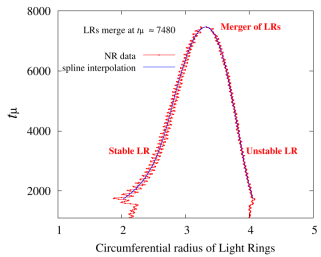


Figure 1: Change of effective potential and radii of light rings.

[a] P. V. P. Cunha and C. Herdeiro, Phys. Rev. Lett. **124**, 181101 (2020);
[b] P. V. P. Cunha, C. Herdeiro, E. Radu and N. Sanchis-Gual, Phys. Rev. Lett. **130**, 061401 (2023).

1. Introduction

geodesic orbits in unstable boson star

- the unstable boson stars will collapse into black holes or migrate into the stable states;
- the initially bounded geodesic orbits will not remain unchanged;
- the nonlinear simulation and numerical computation of $3+1$ geodesics equations are needed.

plan of this work

- perform the nonlinear simulations of three types of concrete spherical boson stars;
- numerically compute the $3+1$ geodesics in the dynamical backgrounds of boson stars;
- obtain the characteristic behaviors of timelike geodesics for observing the gravitational collapse or migration of boson stars.

2. Models and Results

Scalar boson stars (self-interaction $\frac{\lambda}{4}|\Phi|^4$ can stabilize the excited BSs)

We consider the stable, unstable collapsing, and unstable migrating scalar BSs by using the following action [a]

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi^* - V(|\Phi|^2)\right) \tag{1}$$

with $V(|\Phi|^2) = \frac{1}{2}\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4$.

[a] N. Sanchis-Gual, C. Herdeiro and E. Radu, Class. Quant. Grav. **39**, 064001 (2022).

$$\Phi = \phi(r)\exp\left(i\,\omega t\right), \tag{2}$$

$$ds^2 = -e^{2f_0(r)}dt^2 + e^{2f_1(r)}\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]. \tag{3}$$

Here, $f_0(r)$ and $f_1(r)$ are the metric functions, ω is the frequency of the scalar field.

2. Models and Results

three types of boson stars

We consider three types of scalar BSs listed in Table 1.

Table 1: Physical properties and parameters of three types of spherical scalar BSs.

Configuration	$\lambda/(4\pi)$	ω	M_{bs}	fate
BS_a	100	0.92	2.194	***stable***
BS_b	0	0.88	1.357	***collapse***
BS_c	0	0.92	1.284	
BS_d	50	0.96	1.828	***migration***

evolution of system

- The numerical simulations are realized by our own spherical numerical relativity code based on the spherical BSSN formalism;
- The black hole masses are computed using the dynamical apparent horizons framework [a].

[a] A. Ashtekar and B. Krishnan, Living Reviews in Relativity, **7**, 10 (2004).

2. Models and Results

$3+1$ geodesic equations

Combining the spacetime metric in $3+1$ form $\alpha, \beta^i, \gamma^{ij}$, and K^{ij}, we have

$$\frac{dx^i}{dt} = \alpha u^i - \beta^i, \tag{4}$$

$$\frac{du^i}{dt} = \alpha u^j \left(u^i (\partial_j \ln \alpha - K_{jk} u^k) + 2K^i_j - {}^3\Gamma^i_{jk} u^k \right) - \gamma^{ij} \partial_j \alpha - u^j \partial_j \beta^i. \tag{5}$$

initial value of geodesics

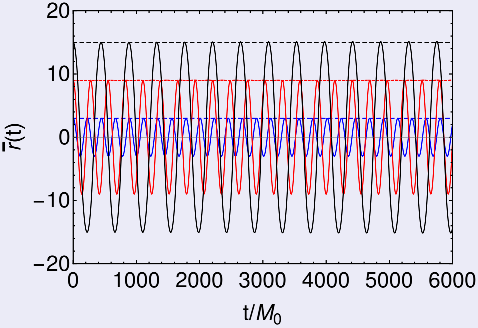
- Boson star is stationary at initial time, and it possesses a timelike killing vector $\xi^\mu = (\partial_t)^\mu$ and a spacelike killing vector $\eta^\mu = (\partial_\varphi)^\mu$;
- We can obtain the initial value by specifying the energy $E = -\xi^\mu u_\mu$ and the angular momentum $J = \eta^\mu u_\mu$.

2. Models and Results

two types of geodesics

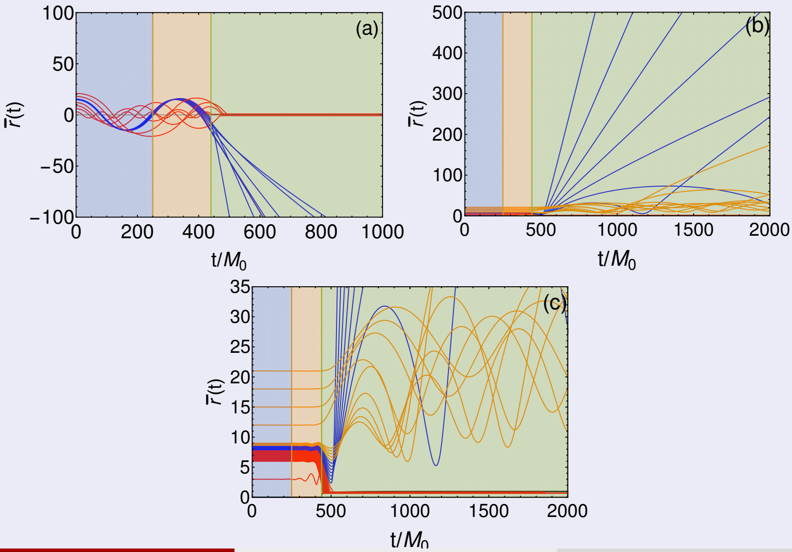
- initially bounded reciprocating geodesic orbits without orbital angular momenta;
- the initially stable circular geodesic orbits with orbital angular momenta.

geodesics in the background of stable boson star BS_a



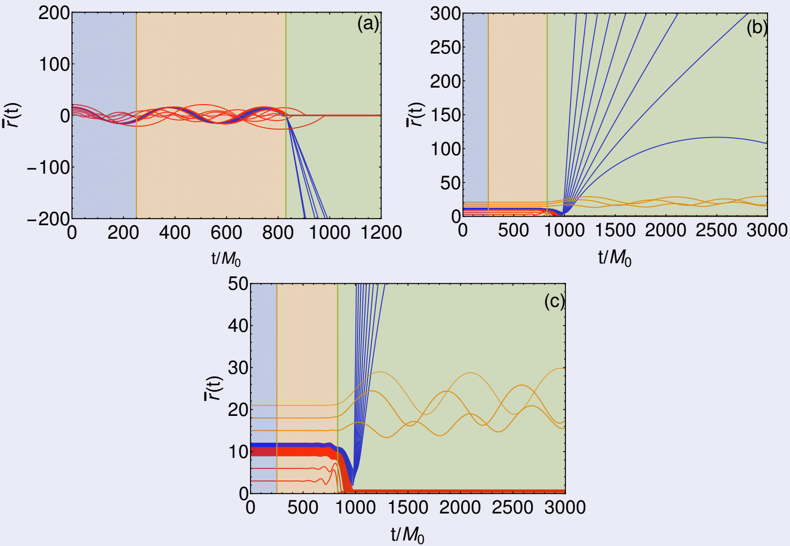
2. Models and Results

geodesics in the background of unstable collapse boson star BS_b



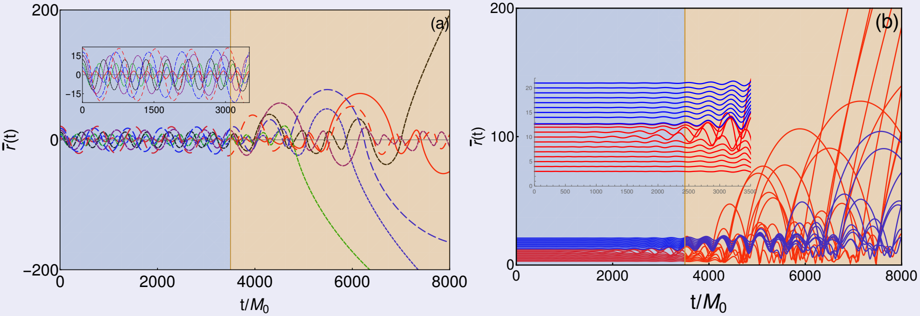
2. Models and Results

geodesics in the background of unstable collapse boson star BS_c



2. Models and Results

geodesics in the background of unstable migration boson star BS_d



3. Summary and Outlook

Summary

We have performed the nonlinear simulations of spherical BSs and investigated the dynamical behaviors of the initially bounded timelike stable circular orbits and reciprocating geodesic orbits.

For unstable collapse boson star:

- plunge into the newly formed black hole;
- escape away from the newly formed black hole;
- stay near the newly formed black hole with a non-zero orbital eccentricity.

For unstable migration boson star:

- escape away from the newly formed boson star;
- stay near the center of boson star with a non-zero orbital eccentricity(initially circular orbits).

Outlook

accreting black hole backgrounds; rotating backgrounds; $\cdots$

Thanks!