Cosmological Constant, Inflaton, Dark Matter Occurred Naturally from Higgs Mechanism for Poincaré Gauge Gravity

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Cosmic inflation, dark matter, and late-time acceleration

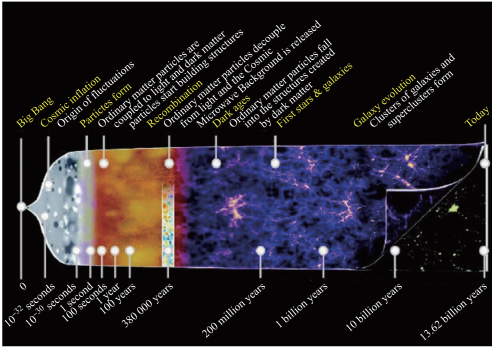

图 1: The cosmic Chronology of the standard model of the universe from the Big Bang to the present epoch. [Bing-Lin Young (2017)]

three tensions between GR and observation, and best solutions

- Cosmic inflation: slow-rolling inflation dominated by a single scalar field
- dark matter: cold dark matter particles
- late-time acceleration: cosmological constant

GR is too pure and perfect to describe gravity, just likes circle orbit. We need conic curves!!

Why gauge theory of gravity?

图 2: Spectrum of the standard model of particle physics

Gauge framework has unified:

Boson	Action	Type	Group
Gluon	Strong	8	$SU(3)$
Photon	EM	1	$U(1)$
W&Z	Weak	3	$SU(2)$

- Symmetries in physics

Global	Local
Internal	External
Abelian	non-Abelian

- Why Poincaré group?

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Nonlinear representation

Let G be a Lie group and H a closed Lie subgroup. A gauge theory of G can be constructed on the principal fibre bundle $G(G/H, H)$. Gauge transformation is defined as automorphism between fibres in a nonlinear way by means of a left action L_g of G on zero sections $\sigma : G/H \to G$ as follows

$$L_g \circ \sigma(\xi) = R_h \circ \sigma(\xi'), \text{ for } g \in G \text{ and } h \in H$$

Then $G/H = \{gH, g \in G\}$ is a homogeneous coset space and can be parameterized by the coset fields ξ. [M. Leclerc (2006)]

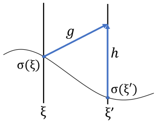

图 3: Automorphism between fibres [E. Lord (1988)]

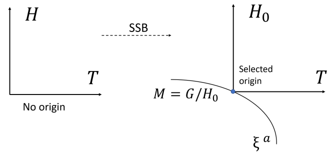

图 4: Spontaneous breakdown of the G group to the H group [I. Kirsch (2005)]

Nonlinear representation of the P-group

$$\text{nonlinear connection:}\quad \tilde{\mathbf{\Gamma}} := \sigma\mathbf{\Gamma}\sigma^{-1} + \sigma \boldsymbol{d}\sigma^{-1}$$

$$\text{P-connection:}\quad \rho(\mathbf{\Gamma})^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} \boldsymbol{A}^{\bar{a}}{}_{\bar{b}} & \boldsymbol{B}^{\bar{a}} \\ 0 & 0 \end{pmatrix}$$

$$\text{coset parameter:}\quad \rho(\sigma)^{\bar{A}}{}_{B} = \begin{pmatrix} \delta^{\bar{a}}_{b} & \xi^{\bar{a}} \\ 0 & 1 \end{pmatrix}$$

$$\text{reduced connection:}\quad \rho(\mathbf{\Gamma})^{A}{}_{B} = \begin{pmatrix} \boldsymbol{A}^{a}{}_{b} & \boldsymbol{\theta}^{a} \\ 0 & 0 \end{pmatrix}$$

$$\text{L-connection:}\quad \boldsymbol{A}^{a}{}_{b} = \delta^{a}_{\bar{a}}\delta^{\bar{b}}_{b}\boldsymbol{A}^{\bar{a}}{}_{\bar{b}}$$

$$\begin{aligned}\text{canonical 1-form:}\quad \boldsymbol{\theta}^{a} &= \delta^{a}_{\bar{a}}(\boldsymbol{d}\xi^{\bar{a}} + \boldsymbol{A}^{\bar{a}}{}_{\bar{b}}\xi^{\bar{b}} + \boldsymbol{B}^{\bar{a}}) = \boldsymbol{d}\xi^{a} + \boldsymbol{A}^{a}{}_{b}\xi^{b} + \boldsymbol{B}^{a} \\ &= \boldsymbol{D}\xi^{a} + \boldsymbol{B}^{a} = \boldsymbol{D}^{P}\xi^{a}\end{aligned}$$

$\boldsymbol{\theta}^a$ can be regarded as the covariant derivatives of ξ^a.

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Cosmological constant

Considering a Higgs field y^a. A L-invariant groundstate $y^a(0)$ that breaks the P-invariance, can only be characterized by a L-invariant relation $\eta_{ab} y^a(0) y^b(0) = 0$, i.e. the state

$$y^a(0) = 0$$

A general state expanded around the groundstate can be parameterized as follows

$$y^a = \xi^a$$

It is pure gauge, i.e. the Nambu-Goldstone bosons. It's nature to set a vanishing potential $V(y) = 0$. The kinetic term

$$e \frac{\lambda}{4} g(\boldsymbol{D}^P y^a, \boldsymbol{D}^P y^b) \eta_{ab} = e \frac{\lambda}{4} g^{\mu\nu} e_\mu{}^a e_\nu{}^b \eta_{ab} = e\lambda$$

with $e_\mu{}^a = \partial_\mu \lrcorner \boldsymbol{\theta}^a$ referred to the tetrad field, and $e \equiv \det(e_\mu{}^a)$.[M. Leclerc (2006)]

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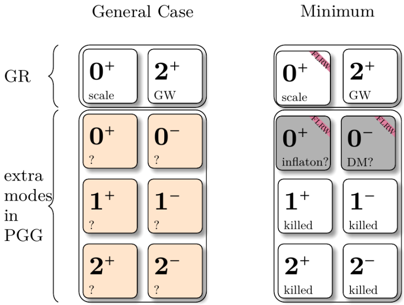
A minimum model

a minimum ghost- and tachyon-free parity-conserving Einstein-Hilbert-Yang-Mills type Lagrangian [HZ and L. Xu (2019), (2020)]

$$
\begin{aligned}
L_G =& b_0 R + \frac{b_0}{3} T_{\mu\nu\rho}(T^{\mu\nu\rho} + T^{\rho\nu\mu} - g^{\mu\rho} T^{\nu}) \\
&+ \frac{2A_1}{3} T_\mu T^\mu + \frac{A_2}{12} T_{\mu\nu\rho}(2T^{\rho\nu\mu} - T^{\mu\nu\rho}) \\
&+ \frac{B_1}{9}(R_{\mu\nu} R^{\nu\mu} - \frac{1}{4} R_{\mu\nu\rho\sigma} R^{\rho\sigma\mu\nu}) \\
&+ \frac{B_2}{9} R_{\mu\nu\rho\sigma}(R^{\mu\rho\nu\sigma} - \frac{1}{4} R^{\mu\nu\rho\sigma} - \frac{1}{4} R^{\rho\sigma\mu\nu})
\end{aligned}
$$

where $b_0 \sim A_1 \sim A_2 \sim m_{Pl}^2$, and B_1, B_2 are dimensionless



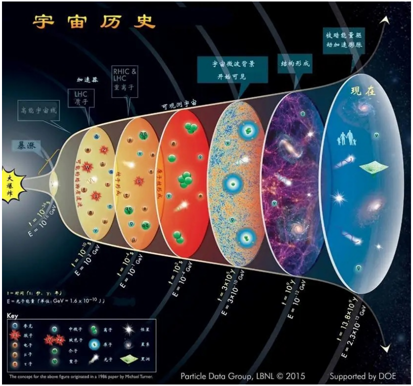
$$\boldsymbol{R}^{ab} := \boldsymbol{d}\boldsymbol{A}^{ab} + \boldsymbol{A}^{a}{}_{c} \wedge \boldsymbol{A}^{c}{}_{b}, \quad \boldsymbol{T}^{a} := \boldsymbol{D}\boldsymbol{\theta}^{a}$$

Cosmological reduction

spatially homogeneous and isotropic reduction, results six global symmetries. All spatial points can be identified, thus forming a zero-dimensional Lie group $\{id\}$ in space. Symmetries referred to the temporal direction are local.

$$
\begin{aligned}
e_0{}^{\hat{0}} &= 1, \quad e_i{}^{\hat{j}} = a(t)\delta_{ij} \\
A_i{}^{\hat{j}\hat{0}} &= a(t)\phi_h(t)\delta_{ij} \\
A_i{}^{\hat{j}\hat{k}} &= -\,a(t)\phi_f(t)\epsilon_{ijk}
\end{aligned}
$$

a replaces t (related to $\xi^{\hat{0}}$) to play the role of Nambu-Goldstone field, i.e. $a \in [0, \infty)$ breaks the symmetry of $\{id\} \times [0, \infty)$ down to $\{id\}$. Any value of a select a freeze-frame of the universe (filled by ϕs). In addition, it's obvious that a is also a pure gauge, therefore a residue Higgs field. A convenient choice of groundstate is $a = 0$.



Massive terms

$$
\begin{aligned}
L_G^0 = &-6b_0(\dot{\phi}_h + H\phi_h - \phi_h^2 + \phi_f^2) - 6A_1(\phi_h + H)^2 - 6A_2\phi_f^2 \\
&+ B_1\left[(\dot{\phi}_h + H\phi_h)^2 - \tfrac{4}{3}(\dot{\phi}_h + H\phi_h)(\phi_h^2 - \phi_f^2)\right. \\
&\left. - \tfrac{4}{3}(\dot{\phi}_f + H\phi_f)\phi_h\phi_f + (\phi_h^2 - \phi_f^2)^2\right] \\
&+ B_2(\dot{\phi}_f + H\phi_f - 2\phi_h\phi_f)^2
\end{aligned}
$$

the Proca masses for ϕ_h and ϕ_f occur in the forms $\frac{1}{2}m_h^2\phi_h^2$, $\frac{1}{2}m_f^2\phi_f^2$

$$
m_h = 2\sqrt{3(A_1 - b_0)}, \quad m_f = 2\sqrt{3(A_2 + b_0)}
$$

Cosmological equations

$$H^2 = \frac{1}{3A_1}\rho_\phi$$
$$2\dot{H} + 3H^2 = -\frac{1}{A_1}p_\phi$$

$$\ddot{\phi}_h + 3H\dot{\phi}_h + (\dot{H} + 2H^2)\phi_h + 2\frac{B_1+B_2}{B_1}\phi_f(\dot{\phi}_f + H\phi_f) + \frac{1}{2B_1}\frac{\partial V_\phi}{\partial \phi_h} + \frac{m_h^2}{2B_1}H = 0$$
$$\ddot{\phi}_f + 3H\dot{\phi}_f + (\dot{H} + 2H^2)\phi_f - 2\frac{B_1+B_2}{B_2}\phi_f(\dot{\phi}_h + H\phi_h) + \frac{1}{2B_2}\frac{\partial V_\phi}{\partial \phi_f} = 0$$

$$\rho_\phi = \frac{B_1}{2}(\dot{\phi}_h + H\phi_h)^2 + \frac{B_2}{2}(\dot{\phi}_f + H\phi_f)^2 + \frac{1}{2}V_\phi$$
$$p_\phi = \frac{1}{3}[\kappa\rho_\phi + \frac{m_h^2}{2}(\dot{\phi}_h + H\phi_h - \phi_h^2) - \frac{m_f^2}{2}\phi_f^2]$$
$$V_\phi = \frac{m_h^2}{2}\phi_h^2 + \frac{m_f^2}{2}\phi_f^2 - B_1(\phi_h^2 - \phi_f^2)^2 - 4B_2\phi_h^2\phi_f^2$$

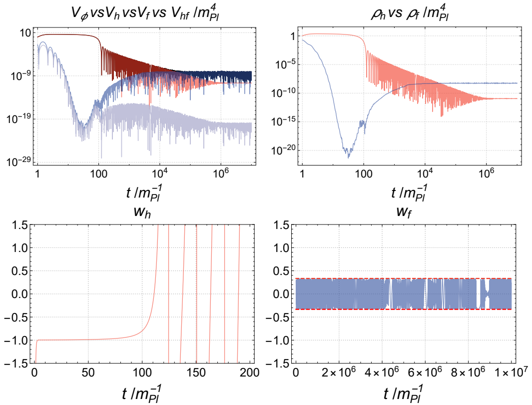
Numerical analysis

test parameters

$$b_0 = 0.01 m_{Pl}^2$$
$$A_1 = A_2 = 1 m_{Pl}^2$$
$$B_1 = B_2 = 1$$

initial conditions

$$\phi_h(0) = \phi_f(0) = 0 m_{Pl}$$
$$\dot{\phi_h}(0) = \dot{\phi_f}(0) = 1 m_{Pl}^2$$



Dark matter candidate

An alternative cold dark matter candidate is a coherently oscillating scalar field, the archetypal example being axion dark matter. Such coherent scalar fields are therefore a well developed alternative to the weakly-interacting massive particle paradigm. [T. Matos and L. A. Urena-Lopez (2001), A. R. Liddle and L. A. Urena-Lopez (2006)]
On the other hand, ϕ_h and ϕ_f are related to the vectorial and axially vectorial components of torsion, respectively

$$T_{i0}{}^{j} = (\phi_h + H)\delta_{ij}, \quad T_{ij}{}^{k} = -2a\phi_f \epsilon_{ijk}$$

These propagating components of torsion can be regarded as the geometric "substances" on the background.
"Torsion cannot propagate" is a misconception brought to us by Einstein-Cartan theory, which is the minimum extension of GR in Riemann-Cartan geometry. In fact, it can be seen that the missing torsion in modern theories of gravity played an important role in the very early universe and dark matter candidates.

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Summary

- We introduce the PGG and the spontaneous symmetry breaking of the P-group down to the L-group in the manner of nonlinear representation, the coset fields ξ^a are pure gauge and play the role of Higgs field
- The kinetic term of the Higgs field can bring the cosmological constant
- We construct a minimum ghost- and tachyon-free parity-conserving Einstein-Hilbert-Yang-Mills type Lagrangian of PGG
- Under the cosmological reduction, the tetrad field residues the scale factor a, and the Lorentz connection residues a scalar field ϕ_h and a pseudo-scalar field ϕ_f
- a breaks the symmetry of $\{id\} \times [0, \infty)$ down to $\{id\}$, and plays the role of Nambu-Goldstone field
- a brings massive terms to ϕ_h and ϕ_f
- The cosmic dynamic is given by the ϕ-sourced and A_1-rescaled Friedmann equations, where ϕ_h dominates a slow-rolling inflation and ϕ_f behaves as a dark matter candidate

In summary, we construct a cosmological model in the framework of PGG, so that cosmological constant, inflaton, and dark matter candidate occur naturally from Higgs mechanism.

Q&A

谢谢!

Thank you for your attention!

Q & A