BLACK HOLE CHEMISTRY

Robert Mann

Black Hole Chemistry Correspondence

Thermodynamics | Gravity

Enthalpy $H \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \frac{\hbar \kappa}{2\pi}$ Surface gravity

Entropy $S \leftrightarrow \frac{A}{4\hbar}$ Horizon Area

Pressure $P \leftrightarrow -\frac{\Lambda}{8\pi}$ Cosmological Constant

First Law

$$\delta M = T_h \delta S_h + \sum_i (\Omega_h^i - \Omega_\infty^i)\delta J^i + \Phi_h \delta Q + V_h \delta P$$

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega_h^i - \Omega_\infty^i) J^i + \frac{D-3}{D-2}\Phi_h Q - \frac{2}{D-2} P V_h$$

The Chemistry of AdS Black Holes

Thermodynamic Potential: Gibbs Free Energy

$$G = M - TS = G(T, P, J_i, Q)$$

- Equilibrium: Global minimum of Gibbs Free Energy
- Local Stability: Positivity of the Specific Heat

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_{P, J_i, Q} > 0$$

Early Results from Black Hole Chemistry

Kubiznak/Mann
CJP **93** 999 (2016)

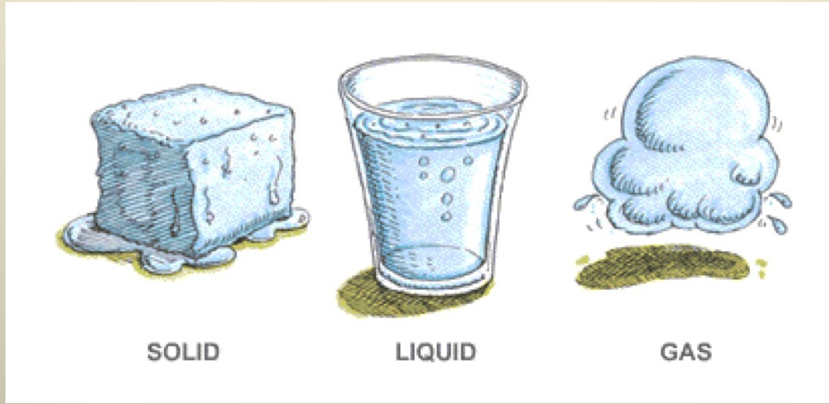
- Hawking Page Transition
  - solid/liquid phase transition with infinite coexistence line

Kubiznak/Mann
JHEP 1207 (2012) 033

- Black Holes as Van der Waals Fluids
  - Complete correspondence between intrinsic and extrinsic variables

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
Galaxies **2** (2014) 89

- Reentrant Phase Transitions
  - Change from one phase to another and back again as one parameter (eg. temperature) monotonically changes
- Black Hole Triple Points ←→ Solid/Liquid/Gas



N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
CQG **31** (2104) 042001

Hawking-Page Transition

S.W. Hawking & D.N. Page
Comm Math. Phys. 87 (1983) 577

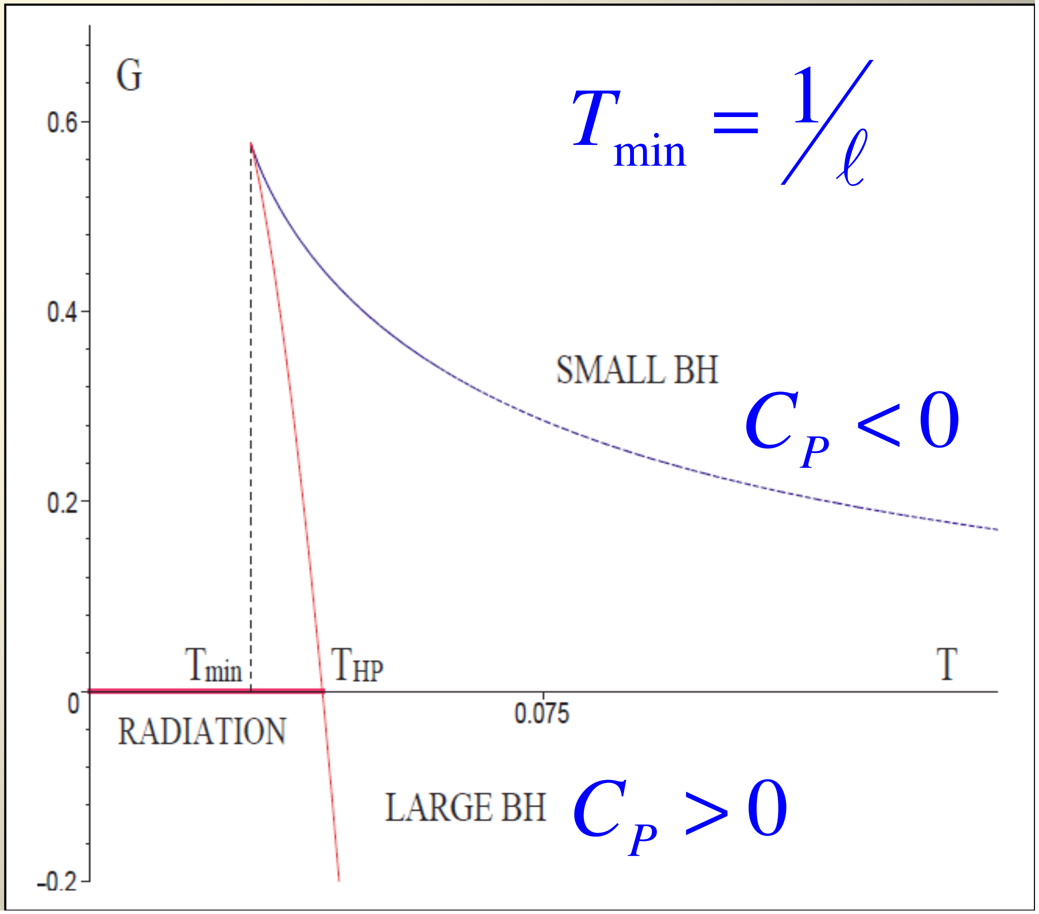
D-dim'l Schwarzschild-AdS Black hole

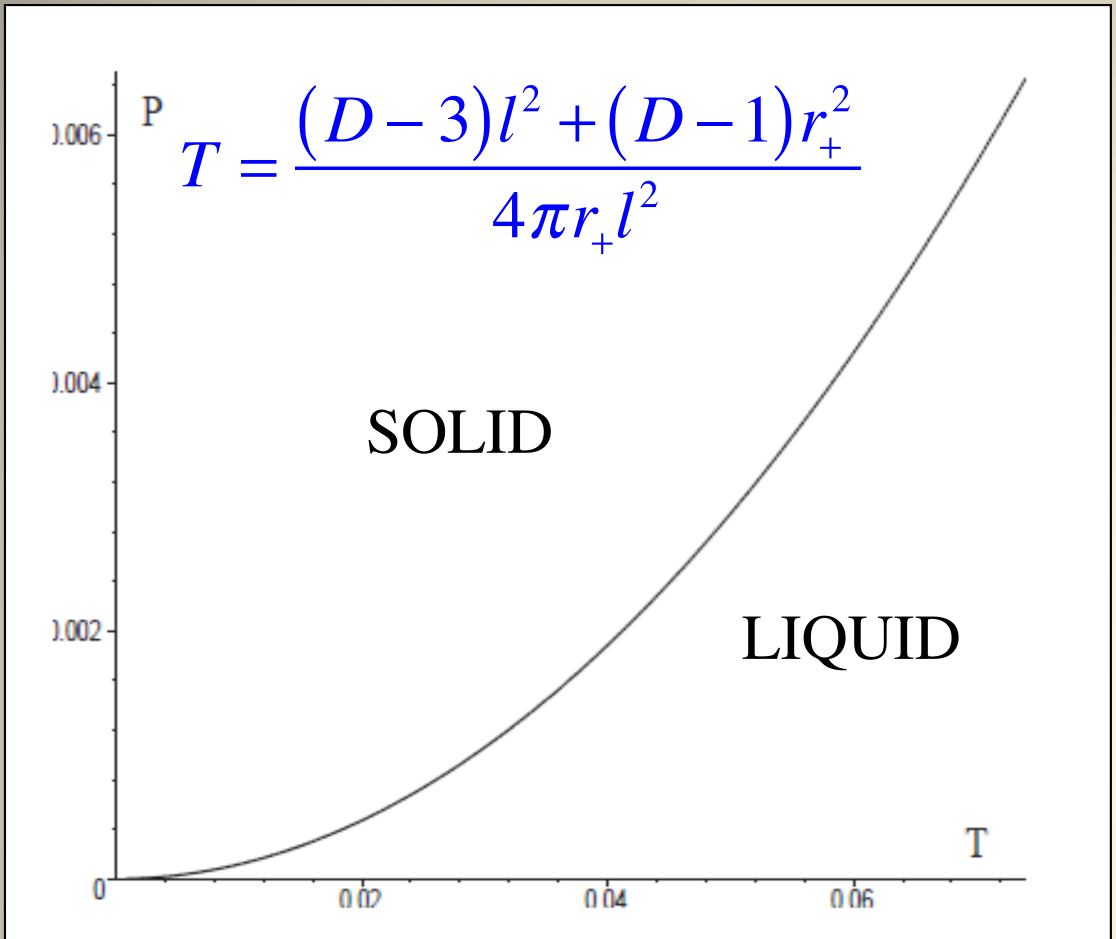
$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega^2_{k,D-2} \qquad V = k - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{\ell^2} \qquad k = \begin{cases} 1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases}$$

- AF black holes evaporate by Hawking radiation
- AdS is like a confining box → static black holes in thermal equilibrium

$$T < T_{\min} \Rightarrow \text{ gas of particles (gravitons?)}$$

1st order transition between gas of particles and large black holes at $T_c = T_{HP}$





Phase transition in dual CFT (quark-gluon plasma)

Witten (1998)

BH Chem interpretation

solid/liquid PT

(infinite coexistence line)

D. Kubiznak/RBM

Can.J.Phys. 93 (2015) 999

Equation of state

$$Pv = T - \frac{k}{2\pi v}$$

depends on the horizon topology

Specific Volume:

$$v = 2l_P^2 r_+ = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 6\frac{V}{N}$$

Volume/DoFs

$$N = \frac{A}{l_P^2}$$

Planar black holes ←→ ideal gas

Van der Waals Fluids

$$\left(P+\frac{a}{v^2}\right)(v-b)=kT$$

$a \rightarrow$ interattractive forces

$b \rightarrow$ finite molecular size

$$Pv^3-(kT+bP)v^2+a(v-b)=0$$

Critical Point

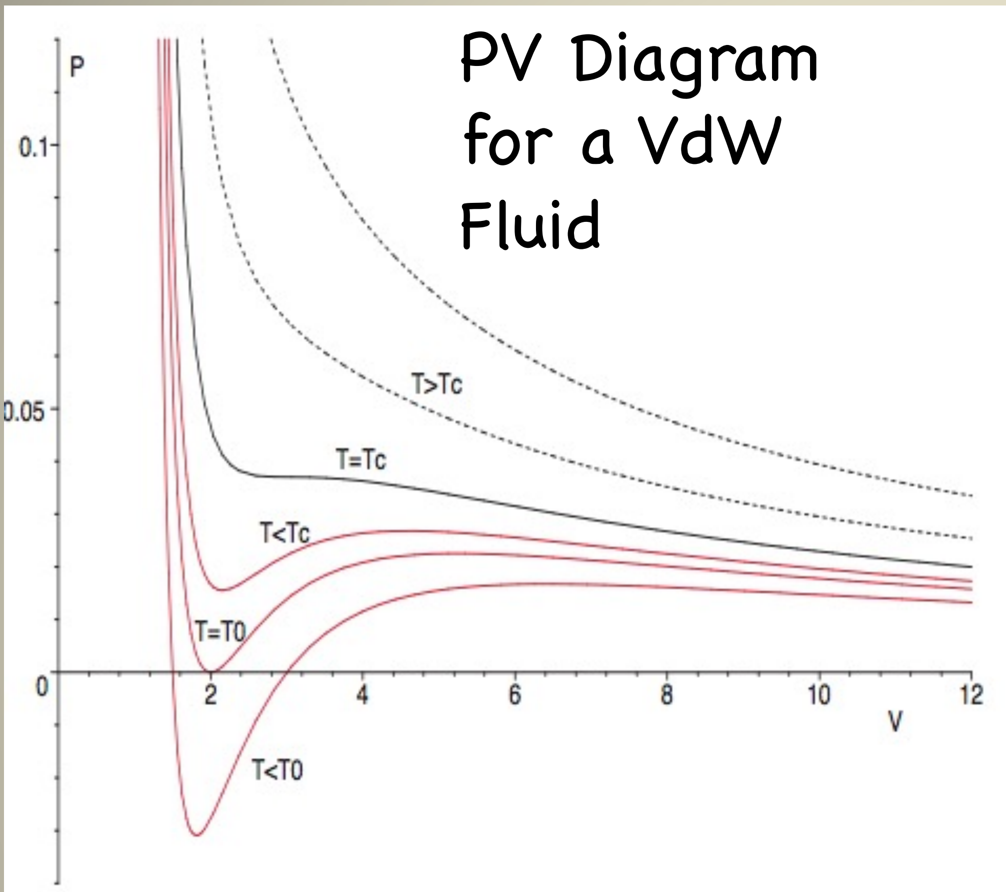
$$\frac{\partial P}{\partial v}=0, \quad \frac{\partial^2 P}{\partial v}=0$$

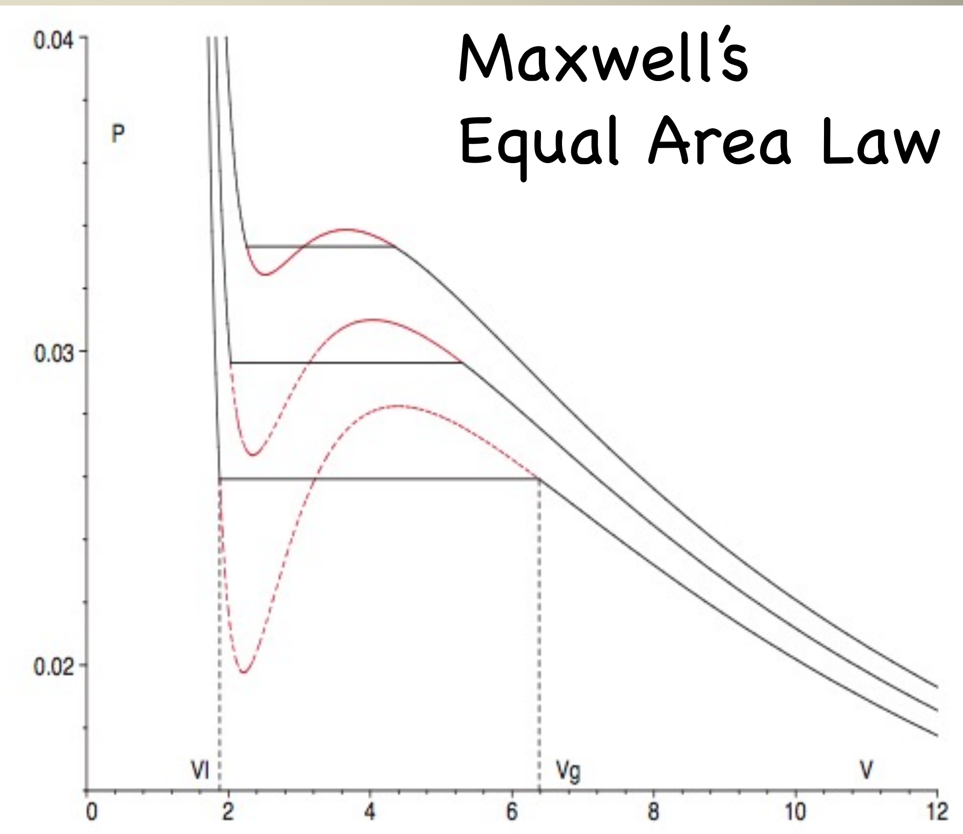
$$kT_c=\frac{8a}{27b}, \quad v_c=3b, \quad P_c=\frac{a}{27b^2}$$

$$p=\frac{P}{P_c}, \quad v=\frac{v}{v_c}, \quad \tau=\frac{T}{T_c}$$

$$8\tau=(3v-1)\left(p+\frac{3}{v^2}\right)$$

law of corresponding states





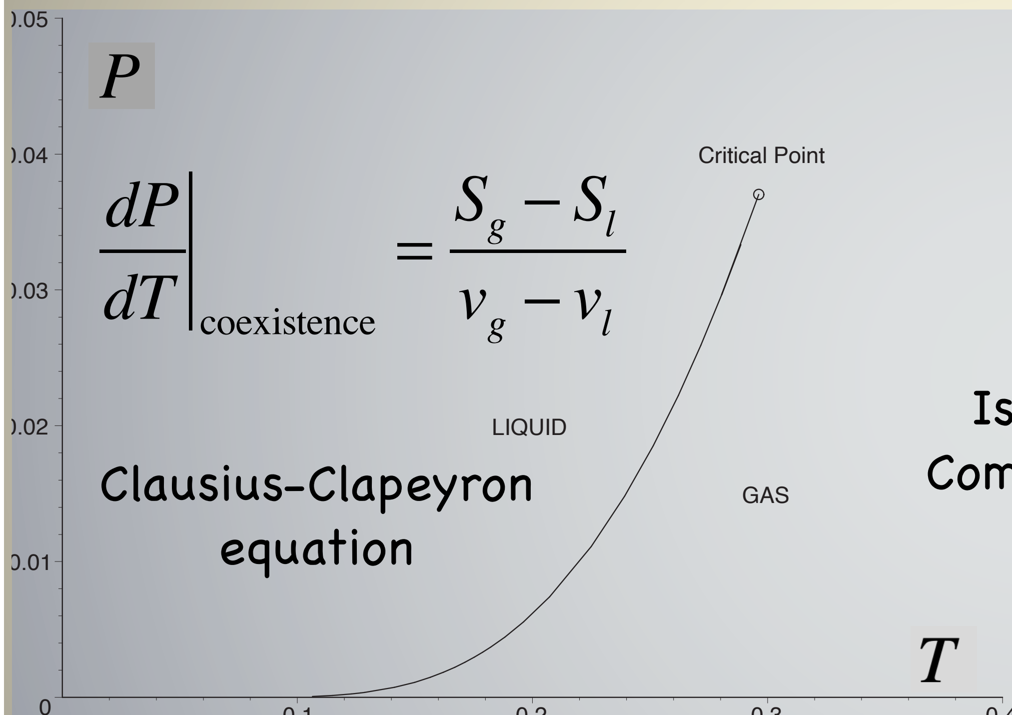
$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

$$\oint v dP = 0$$

Critical Exponents

$$p=\frac{P}{P_c},\quad v=\frac{v}{v_c},\quad \tau=\frac{T}{T_c}$$

Line of Coexistence



$$\left.\frac{dP}{dT}\right|_{\text{coexistence}}=\frac{S_g-S_l}{v_g-v_l}$$

Clausius-Clapeyron equation

Specific Heat
$$C_v=T\left.\frac{\partial S}{\partial T}\right|_v \propto|\tau-1|^{-\alpha}$$

Order Parameter
$$\eta=v_g-v_l\propto|\tau-1|^{\beta}$$

Isothermal Compressibility
$$\kappa_T=-\frac{1}{v}\left.\frac{\partial v}{\partial P}\right|_T \propto|\tau-1|^{-\gamma}$$

Critical Isotherm
$$|P-P_c|\propto|v-v_c|^{\delta}$$

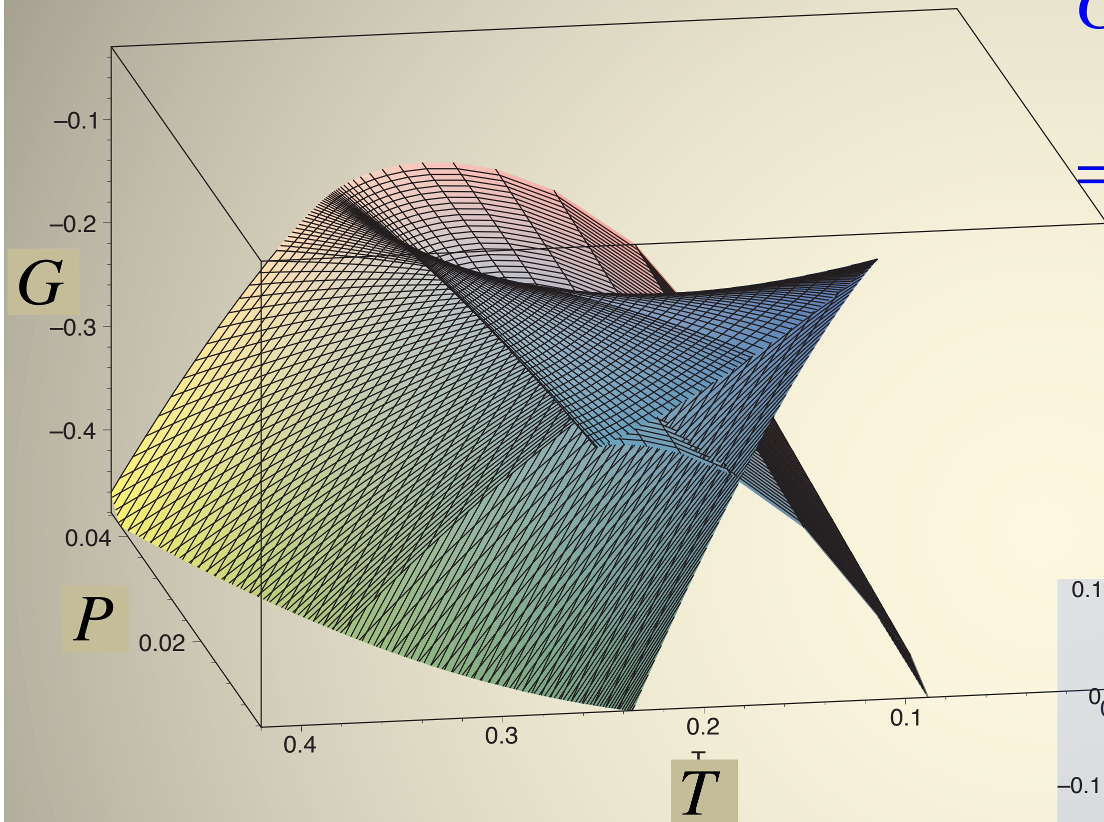
For a VdW Fluid $\alpha=0\quad \beta=\frac{1}{2}\quad \gamma=1\quad \delta=3$

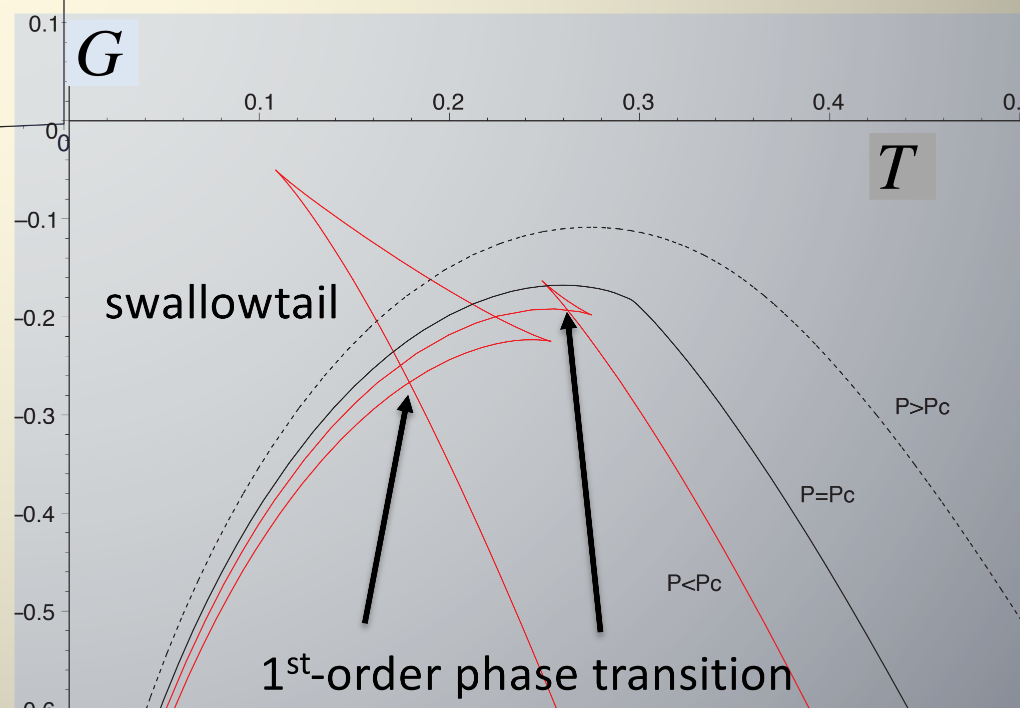
Gibbs Free Energy

$$G = G(T, P)$$

$$= -kT\left(1 + \ln\left[\frac{(v-b)T^{3/2}}{\Phi}\right]\right) - \frac{a}{v} + Pv$$

characteristic of the gas





First Law

$$dG = -S\,dT + v\,dP$$

Charged AdS Black Holes as Van der Waals Fluids

Kubiznak/Mann
JHEP 1207
(2012) 033

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \qquad F = dA \quad \begin{cases} f = 1 - \dfrac{2M}{r} + \dfrac{Q^2}{r^2} + \dfrac{r^2}{l^2} \\ A = -\dfrac{Q}{r}dt \end{cases}$$

Temperature $\quad T = \frac{1}{\beta} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$

Entropy $\quad S = \frac{A}{4} = \pi r_+^2$ $\qquad$ Pressure $\quad P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}$

Potential $\quad \Phi = \frac{Q}{r_+}$ $\qquad$ Volume $\quad V = \frac{4}{3}\pi r_+^3$

$$M = 2\left(TS - PV\right) + \Phi Q$$

Smarr Relation

$$dM = TdS + VdP + \Phi dQ$$

First Law

Equation of State

$$r_+ = \left(\frac{3V}{4\pi}\right)^{1/3}$$

$$T = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$$

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}$$

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}$$

Physical Equation of State

Thermodynamic Specific Volume

$$\text{Press} = \frac{\hbar c}{l_P^2}P \quad \text{Temp} = \frac{\hbar c}{k_B}T \quad \Rightarrow \quad \text{Press} = \frac{k_B \text{Temp}}{2l_P^2 r_+} + \ldots \quad \Rightarrow \quad v = 2l_P^2 r_+$$

Van der Waal's Equation

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

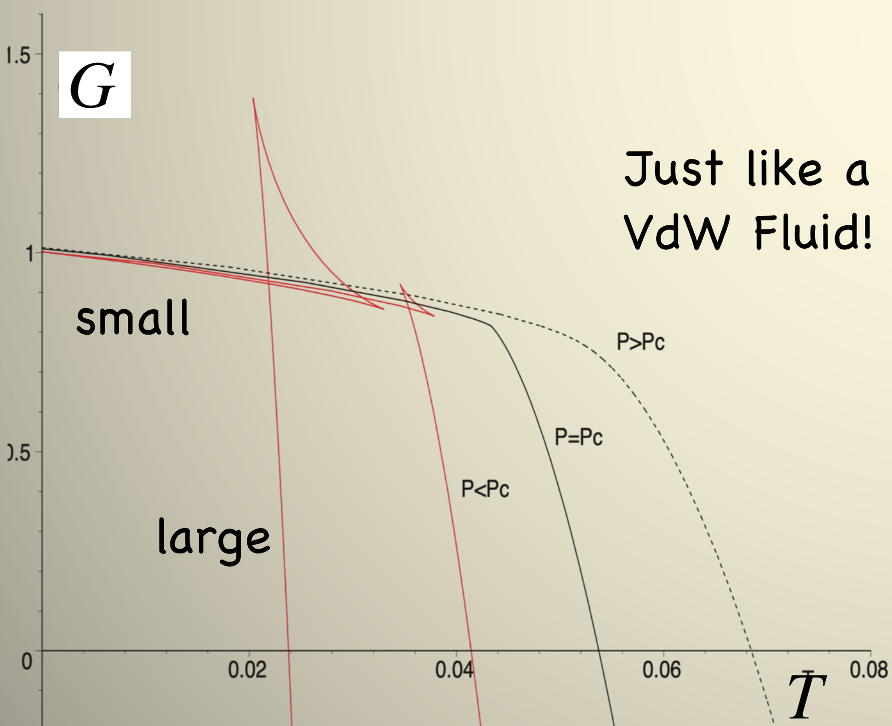
Gibbs Free Energy of AdS RN BH

$$I = -\frac{1}{16\pi}\int_M \sqrt{-g}\left(R - F^2 + \frac{6}{l^2}\right) - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}K - \frac{1}{4\pi}\int_{\partial M} d^3x\sqrt{h}n_a F^{ab}A_b + I_c$$

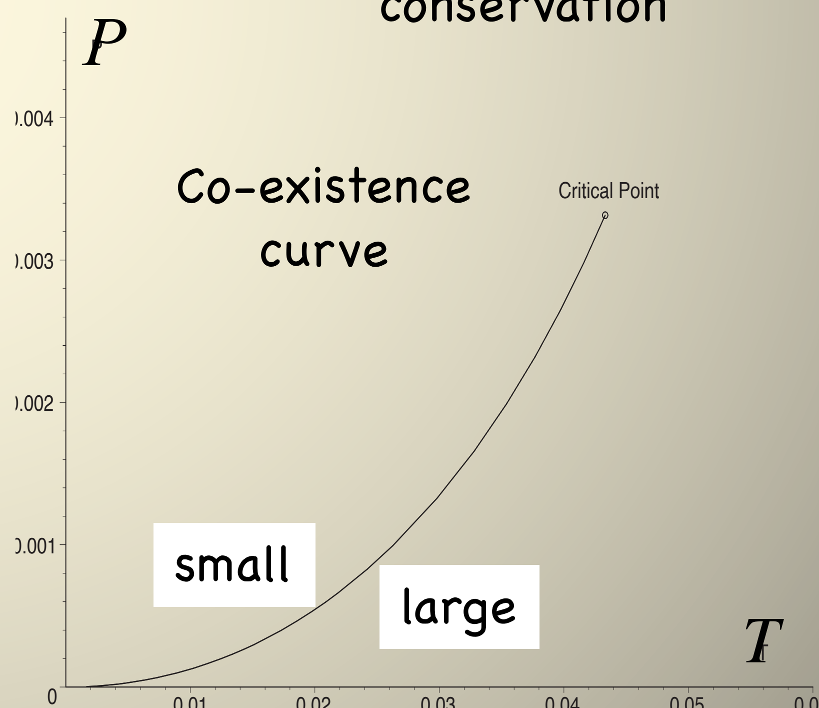
Fixed Charge

$$G = G(T,P) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$

No transition to thermal radiation due to charge conservation



Just like a VdW Fluid!



Critical Behaviour

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

$$\frac{\partial P}{\partial v} = 0 \quad \frac{\partial^2 P}{\partial v^2} = 0$$

$$T_c = \frac{\sqrt{6}}{18\pi Q}$$

$$v_c = 2\sqrt{6}Q$$

$$P_c = \frac{1}{96\pi Q^2}$$

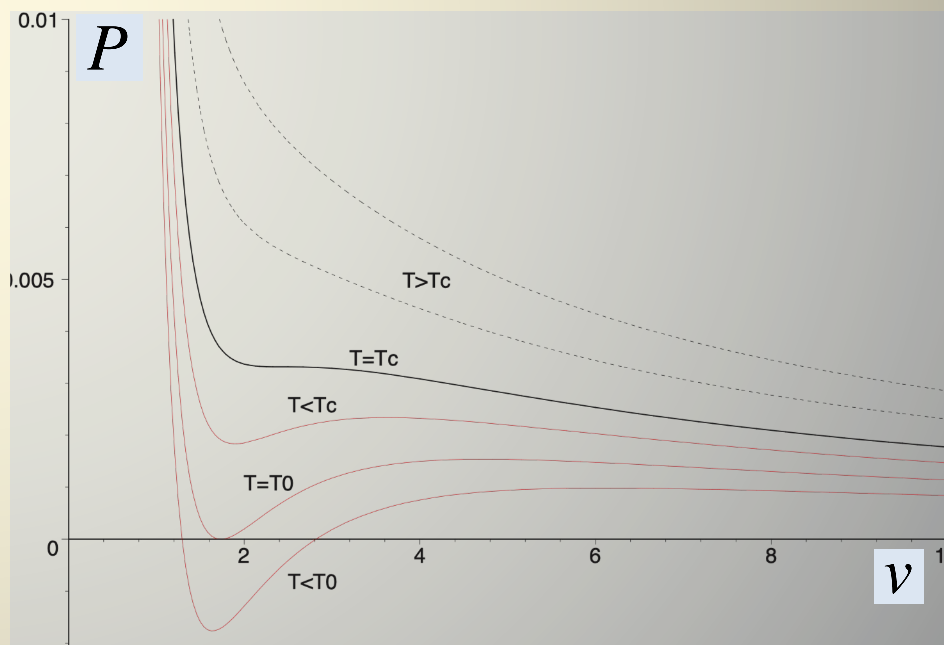
$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

Just like a VdW Fluid!

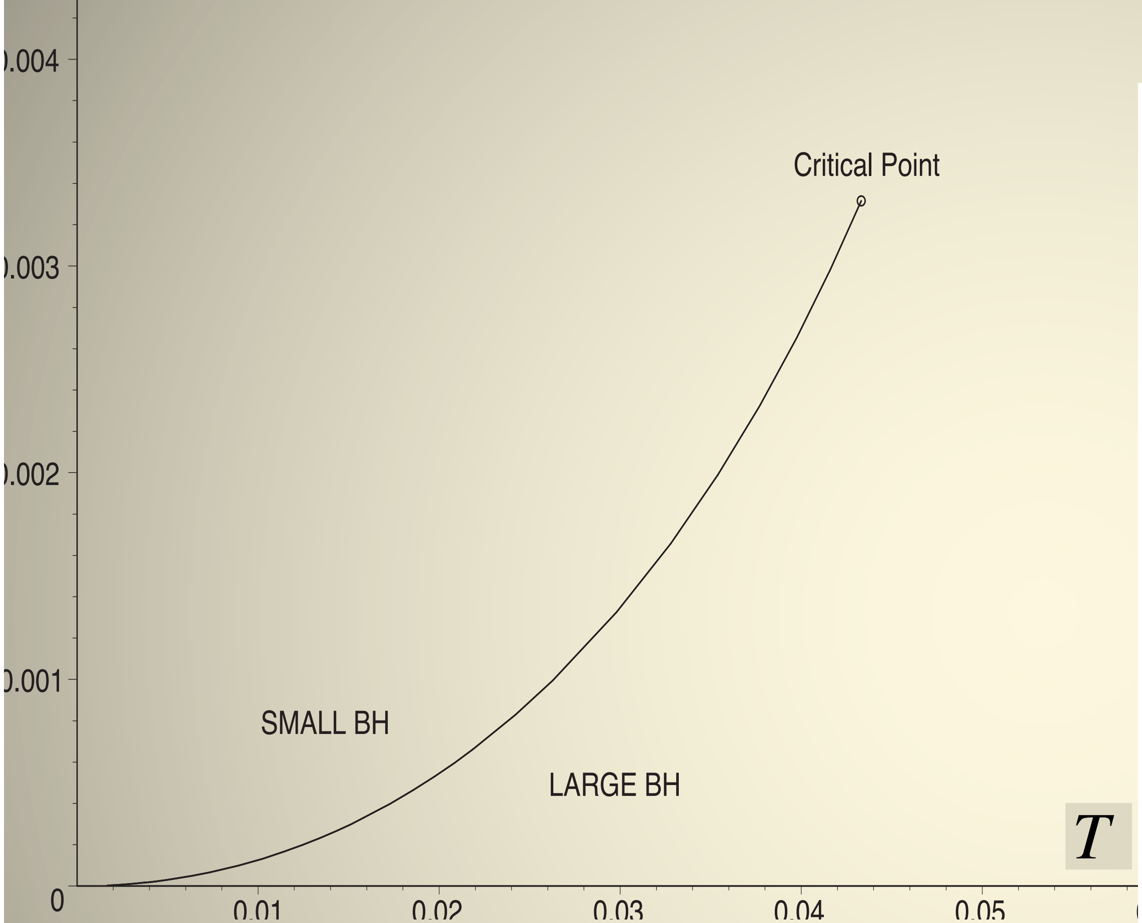
$$p = \frac{P}{P_c}, \quad v = \frac{v}{v_c}, \quad \tau = \frac{T}{T_c}$$

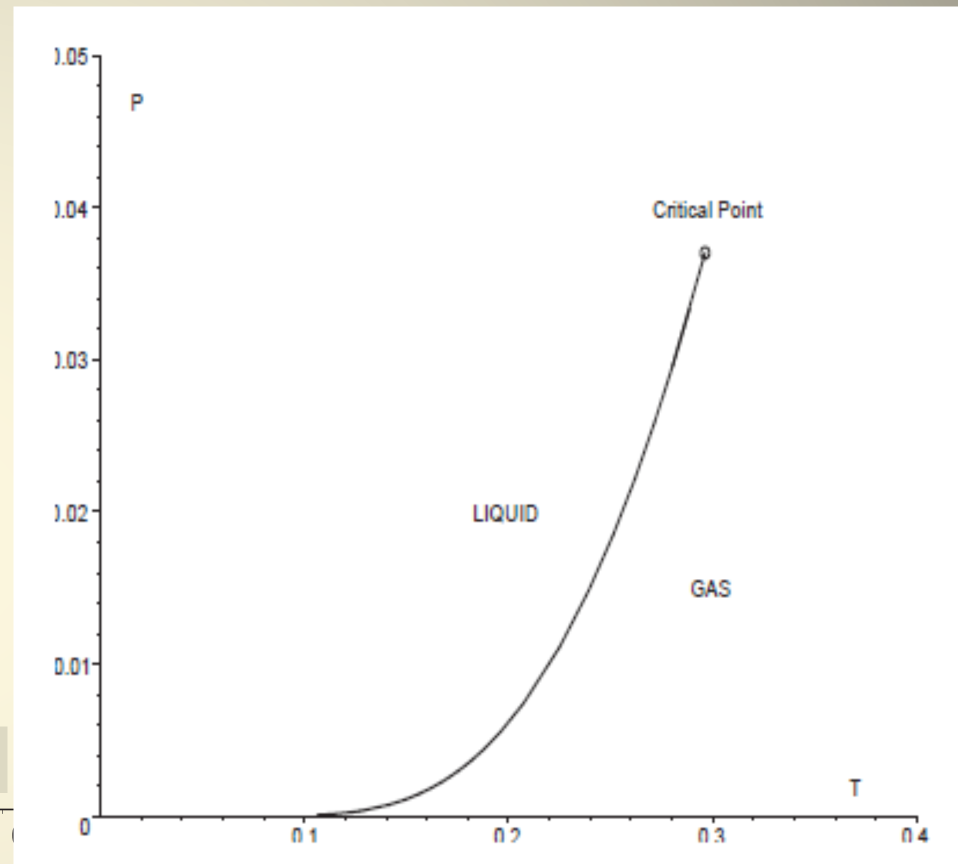
$$8\tau = 3v\left(p + \frac{2}{v^2}\right) - \frac{1}{v^3}$$

law of corresponding states



Just like a VdW Fluid!





$$\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3 \longrightarrow$$ Mean Field Theory

govern volume, compressibility, specific heat, and pressure near the critical point

Previous VdW BH Analogies

Chamblin et.al. PRDD60 (1999) 064018; 104026

Analogy 1: Canonical	
Fluid	AdS Black Hole
Temperature	Q
Pressure	$1/T$
Volume	r_+

Analogy 2: Grand Canonical	
Fluid	AdS Black Hole
Temperature	$1/T$
Pressure	Q
Volume	Φ

Problems: (a) Confusion of intensive and extensive quantities

(b) Minimal temperature below which phase transition doesn't occur

Kubiznak/Mann
JHEP 1207 (2012) 033

Black Hole Chemistry	
Fluid	AdS Black Hole
Temperature	T
Pressure	$-\Lambda / 8\pi G$
Volume	$4\pi r_+^3 / 3$

Analogy Complete!

Charged D-Dim'l AdS Black Holes

Gunasekaran/Kubiznak/Mann
JHEP 1211 (2012) 110

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2_{D-2} \qquad F = dA \quad \begin{cases} f = 1 - \dfrac{2M}{r^{D-3}} + \dfrac{q^2}{r^{2(D-3)}} + \dfrac{r^2}{l^2} \\ A = -\sqrt{\dfrac{D-2}{2(D-3)}}\dfrac{q}{r} \end{cases}$$

$$M = \frac{D-2}{16\pi}\omega_{D-2} m$$

$$Q = \frac{\sqrt{2(D-2)(D-3)}}{8\pi}\omega_{D-2}\, q$$

Temperature $T = \dfrac{D-3}{4\pi r_+}\left(1 - \dfrac{q^2}{r_+^{2(D-3)}} + \dfrac{D-1}{D-3}\dfrac{r_+^2}{l^2}\right)$

Entropy $S = \dfrac{A_{D-2}}{4} = \dfrac{\omega_{D-2} r_+^{D-2}}{4}$ Pressure $P = -\dfrac{\Lambda}{8\pi} = \dfrac{(D-1)(D-2)}{16\pi l^2}$

Potential $\Phi = \sqrt{\dfrac{D-2}{2(D-3)}}\dfrac{q}{r_+^{D-3}}$ Volume $V = \dfrac{\omega_{D-2} r_+^{D-1}}{D-1}$

$$\omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma[(D-1)/2]}$$

$$M = \frac{D-2}{D-3}TS - \frac{2}{D-3}VP + \Phi Q \qquad dM = TdS + VdP + \Phi dQ$$

Smarr Relation First Law

Equation of State

$$r_+ = \left(\frac{(D-1)V}{4\pi}\right)^{1/(D-1)}$$

$$T = \frac{D-3}{4\pi r_+}\left(1 - \frac{q^2}{r_+^{2(D-3)}} + \frac{D-1}{D-3}\frac{r_+^2}{l^2}\right)$$

$$P = \frac{(D-1)(D-2)}{16\pi l^2}$$

$$P = \frac{T(D-2)}{4r_+} - \frac{(D-3)(D-2)}{16\pi r_+^2} + \frac{q^2(D-3)(D-2)}{16\pi r_+^{2(D-2)}}$$

Physical Equation of State

Specific Volume

$$\text{Press} = \frac{\hbar c P}{l_P^{d-2}} \quad \text{Temp} = \frac{\hbar c T}{k_B}$$

$$\text{Press} = \frac{k_B(D-2)\text{Temp}}{4 l_P^{d-2} r_+} + \ldots$$

$$v = \frac{2 l_P^{D-2} r_+}{D-2}$$

Van der Waal's Equation

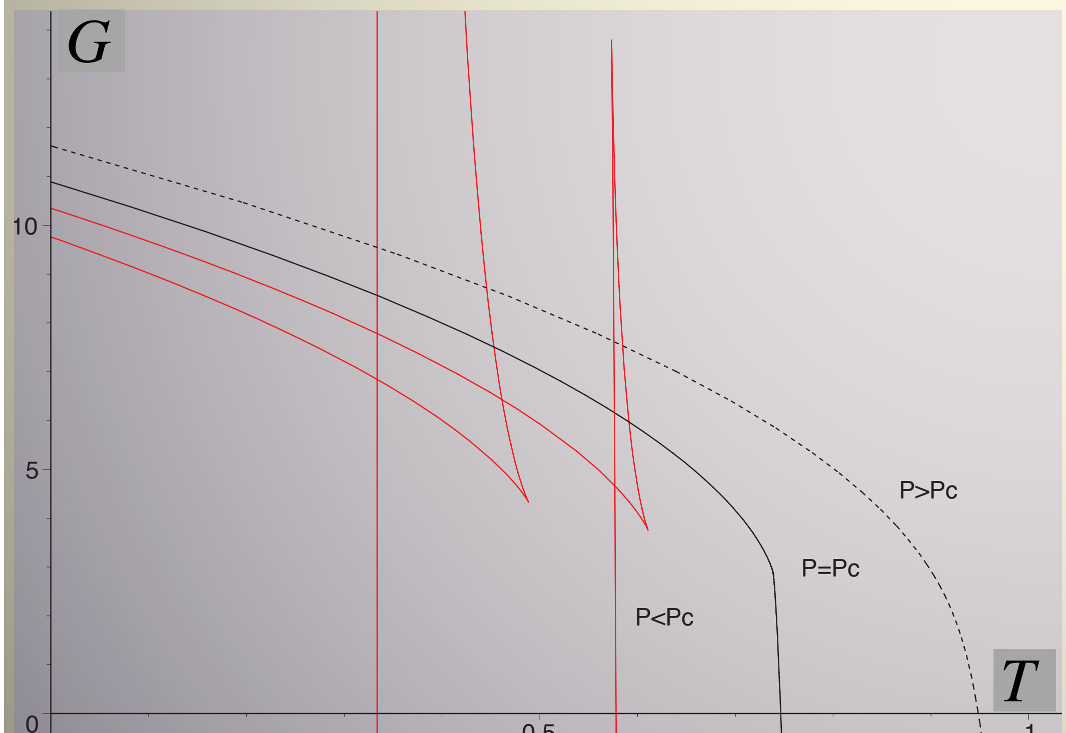
$$P = \frac{T}{v} - \frac{D-3}{(D-2)\pi v^2} + \frac{(D-3)q^2}{4\pi v^{2(D-2)}\kappa^{2D-5}}$$

$$r_+ = \kappa v = \frac{D-2}{2} v$$

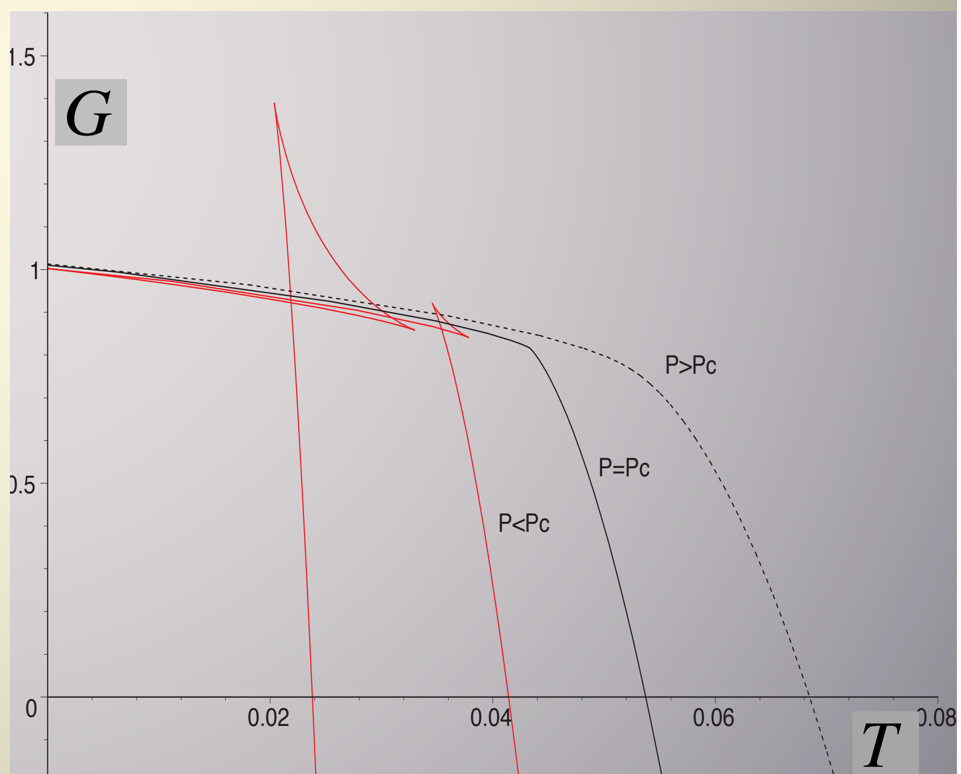
Gibbs Free Energy of D-Dim AdS RN BH

$$G = G(T,P) = \frac{\omega_{D-2}}{16\pi}\left(r_+^{D-3} - \frac{16\pi P r_+^{D-1}}{(D-1)(D-2)} + \frac{(2D-5)q^2}{r_+^{D-3}} \right)$$

10 Dimensions



4 Dimensions



Critical Behaviour

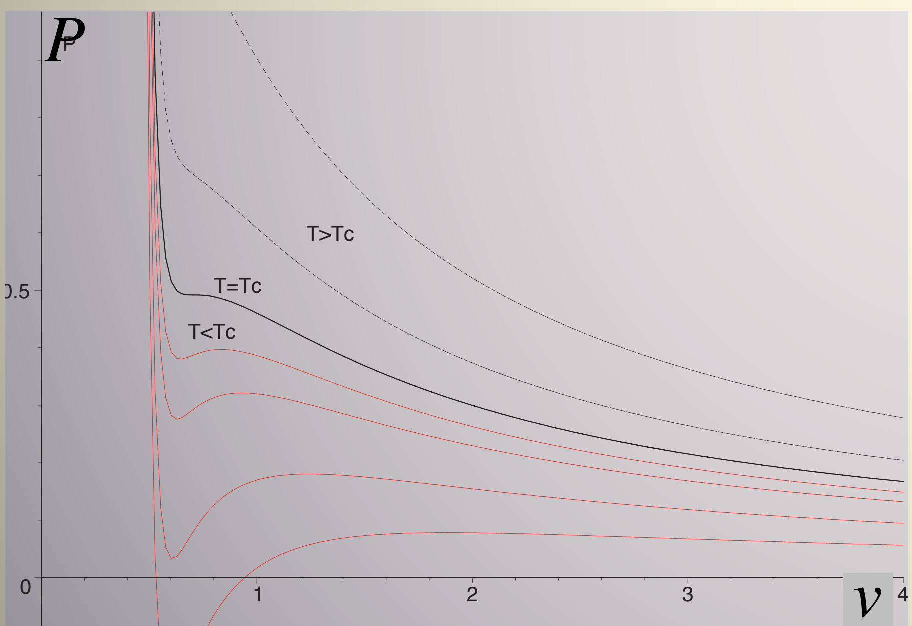
$$P=\frac{T}{v}-\frac{D-3}{(D-2)\pi v^2}+\frac{(D-3)q^2}{4\pi v^{2(D-2)}\kappa^{2D-5}}$$

$$\frac{\partial P}{\partial v}=0 \quad \frac{\partial^2 P}{\partial v^2}=0$$

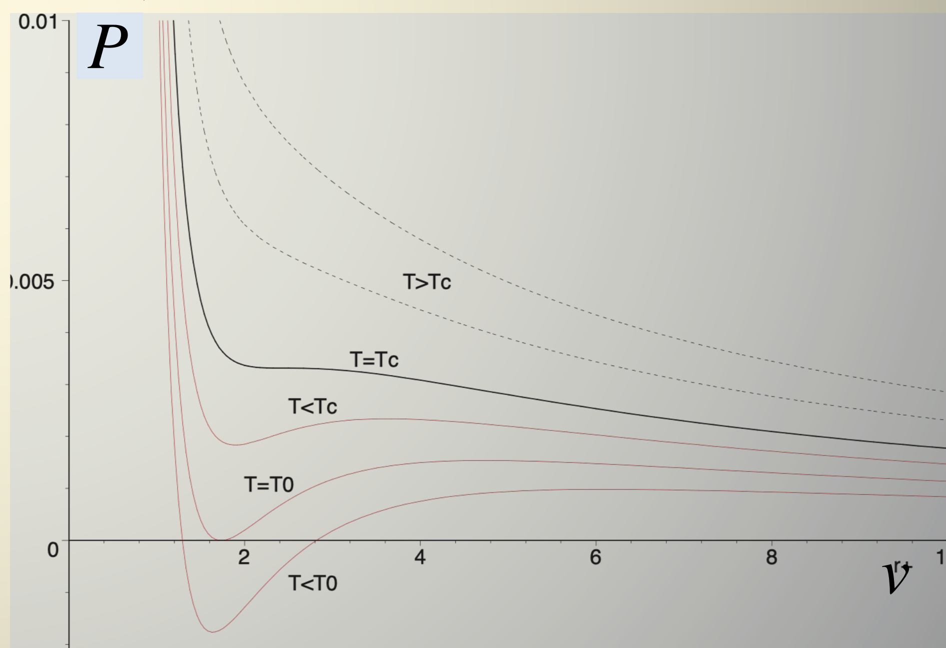
$$\frac{P_c v_c}{kT_c}=\frac{2D-5}{4D-8}$$

$$4(D-2)\tau=(2D-5)v\left(p+\frac{D-2}{(D-3)v^2}\right)-\frac{1}{(D-3)v^3}$$

law of corresponding states



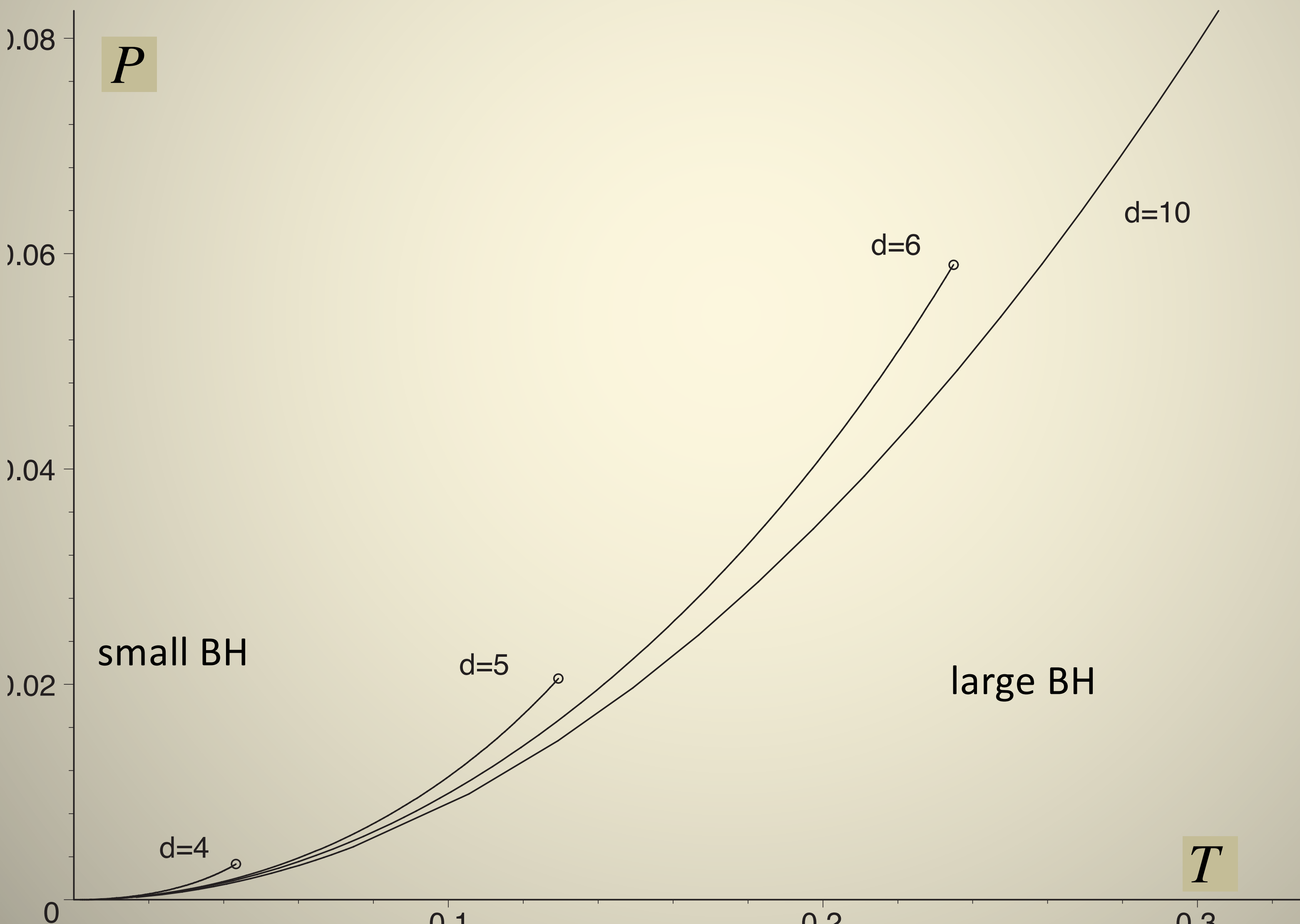
10 Dimensions



4 Dimensions

$$\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3$$

Just like a
4d- VdW
Fluid!

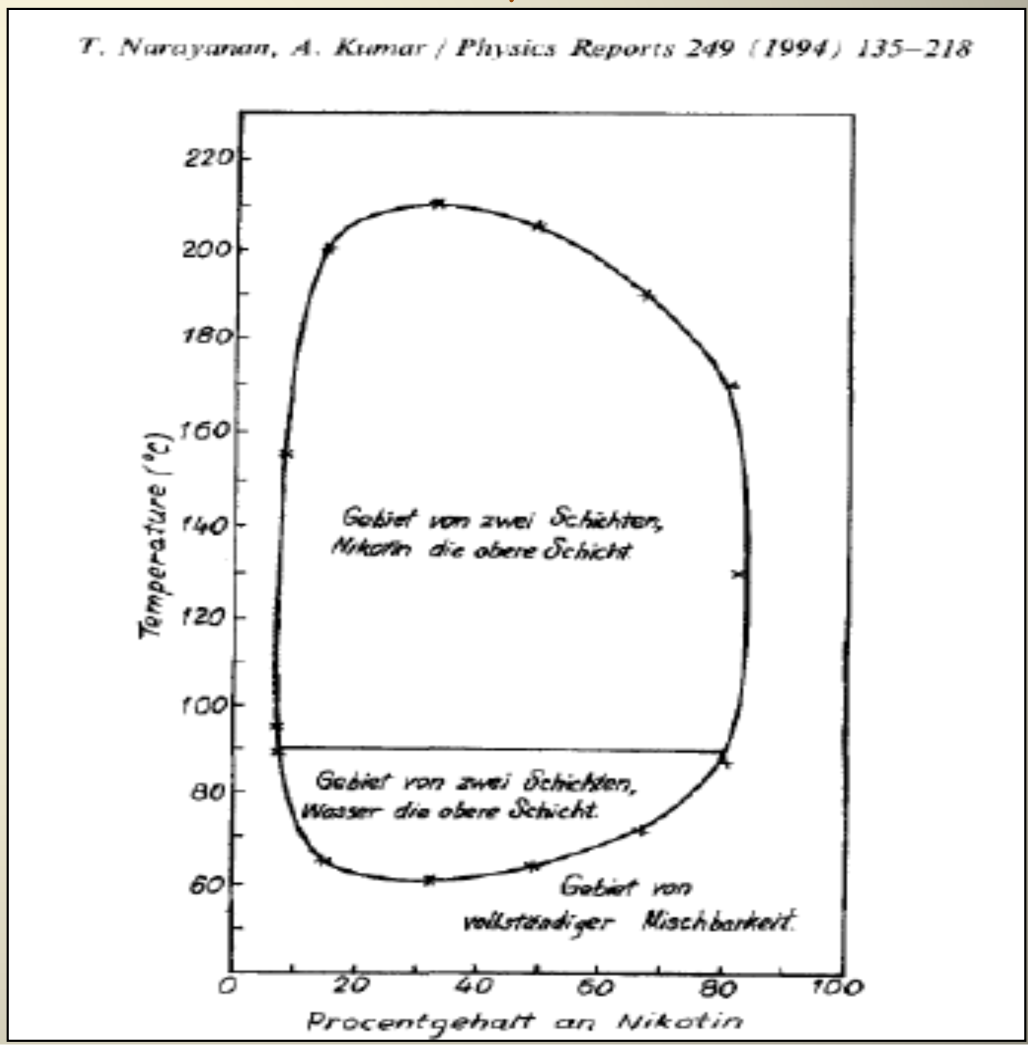


Re-entrant Phase Transitions

A system undergoes an RPT if a **monotonic** variation of any thermodynamic quantity results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

C. Hudson
Z. Phys. Chem. 47 (1904) 113.

First observed in nicotine/water



And later in many other systems:

- multicomponent fluid systems
- gels
- ferroelectrics
- liquid crystals
- binary gases

T. Narayanan and A. Kumar
Physics Reports 249 (1994) 135

And in Black Holes!

Single-Rotation Black Holes

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - \frac{a\sin^2\theta d\varphi}{\Xi}\right]^2 + \frac{\Sigma\sin^2\theta}{\rho^2}\left[adt - \frac{\left(r^2+a^2\right)d\varphi}{\Xi}\right]^2$$

$$+\frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Sigma}d\theta^2 + r^2\cos^2\theta d\Omega_{D-2}$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Sigma = 1 - \frac{a^2}{l^2}\cos^2\theta \qquad \Xi = 1 - \frac{a^2}{l^2}$$

$$\Delta = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2mr^{5-D}$$

$$M = \frac{\omega_{D-2}}{4\pi}\frac{m}{\Xi^2}\left(1+\frac{(D-4)\Xi}{2}\right) \qquad J = \frac{\omega_{D-2}}{4\pi}\frac{ma}{\Xi^2} \qquad \Omega_H = \frac{a}{l^2}\frac{r_+^2+l^2}{r_+^2+a^2},$$

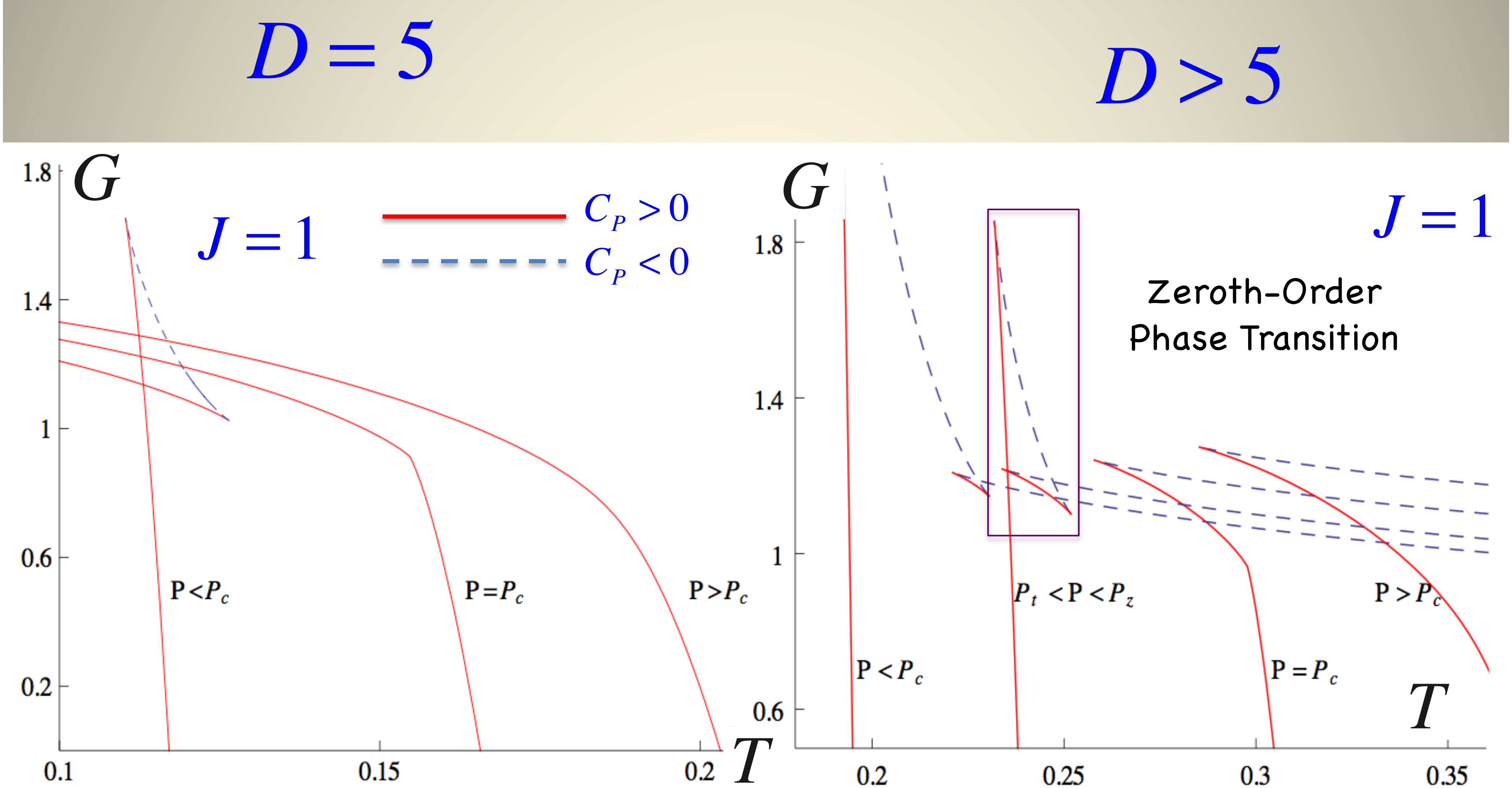
$$T = \frac{1}{2\pi}\left[r_+\left(\frac{r_+^2}{l^2}+1\right)\left(\frac{1}{a^2+r_+^2}+\frac{D-3}{2r_+^2}\right) - \frac{1}{r_+}\right] \qquad S = \frac{\omega_{D-2}}{4}\frac{(a^2+r_+^2)r_+^{d-4}}{\Xi} = \frac{A}{4}$$

$$\frac{D-3}{D-2}M = TS + \Omega_H J - \frac{2VP}{D-2} \qquad dM = TdS + VdP + \Omega_H dJ$$

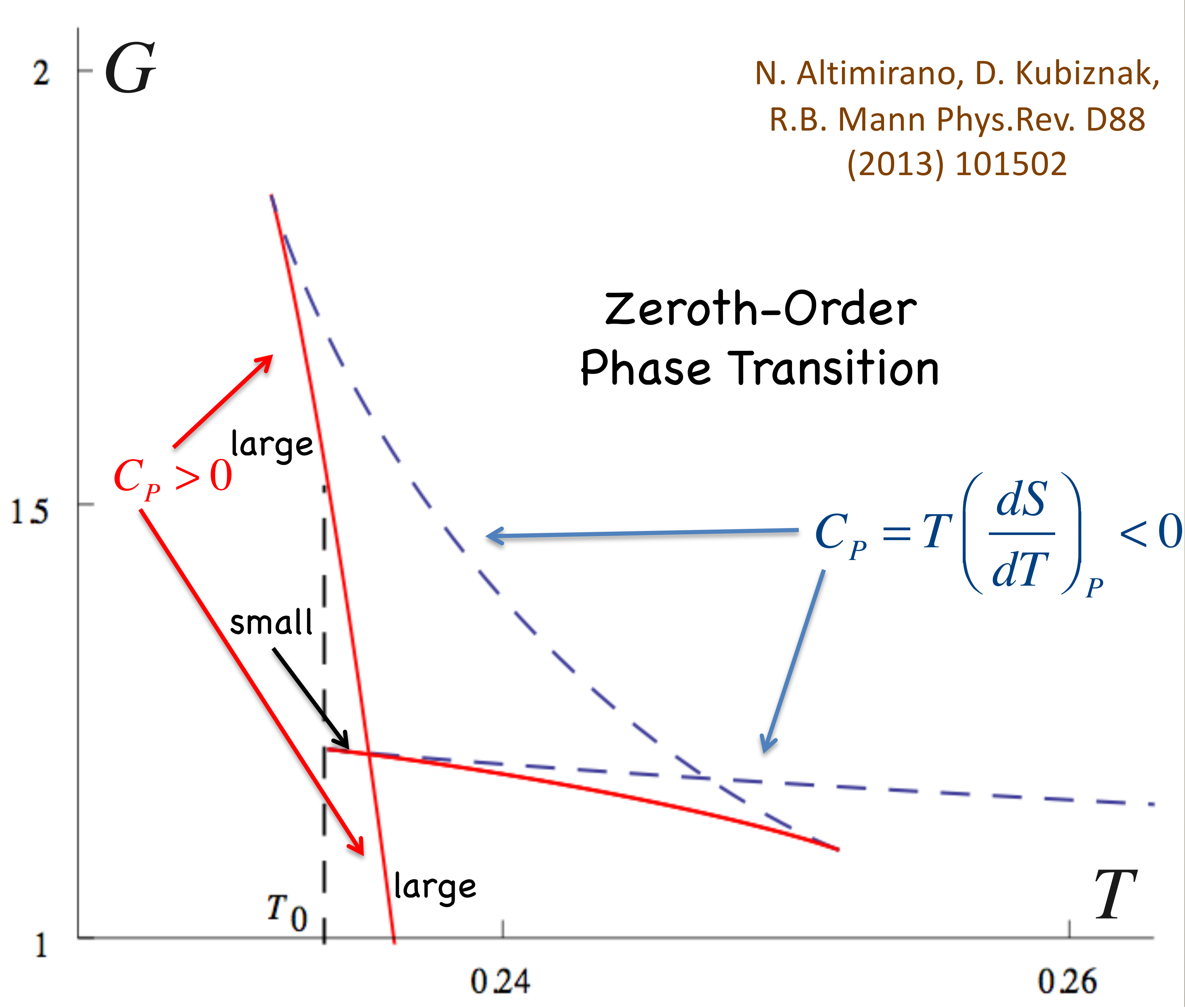
Smarr Relation

First Law

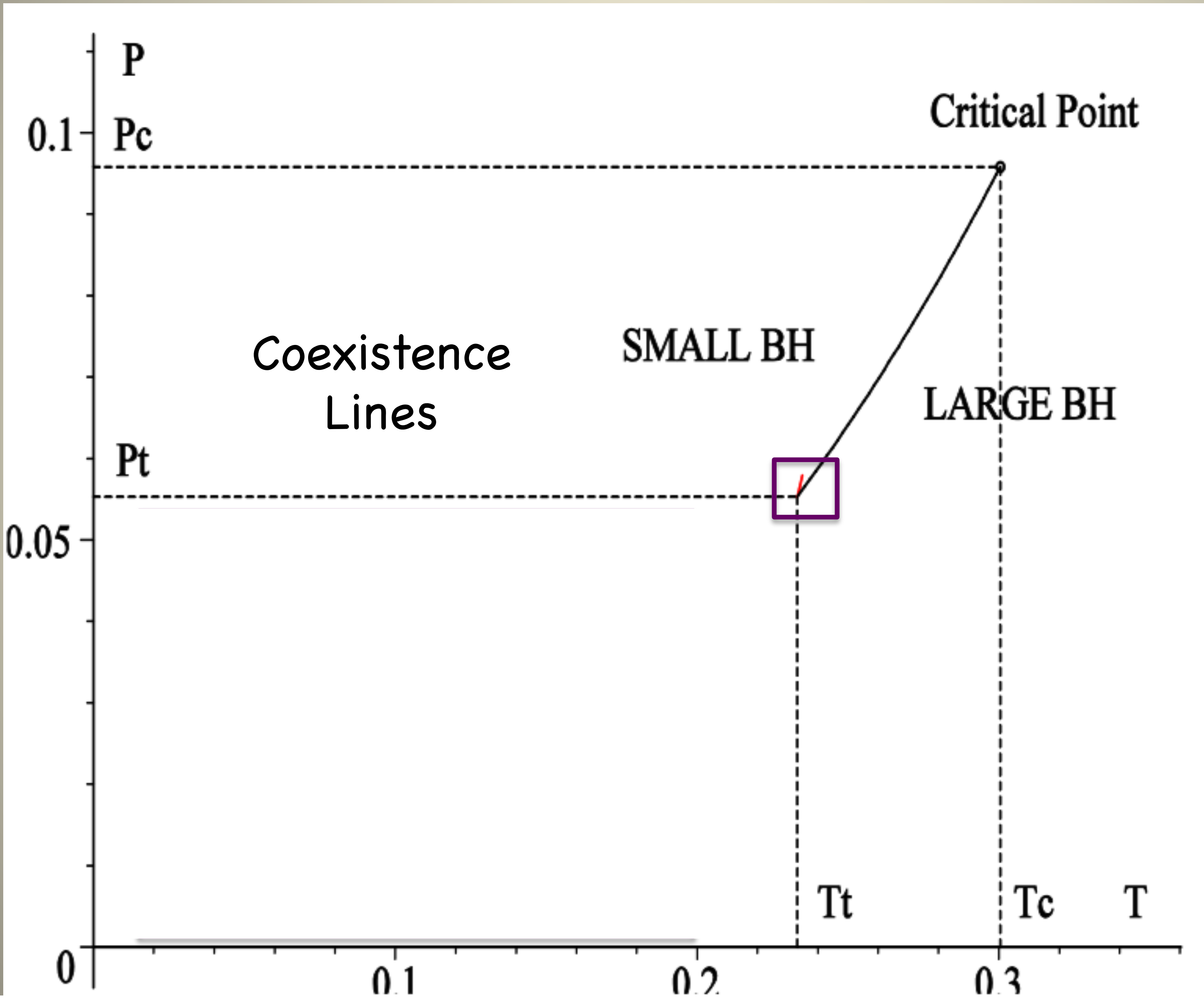
Dimensional Dependence of G



Reentrant Phase Transitions in D>5



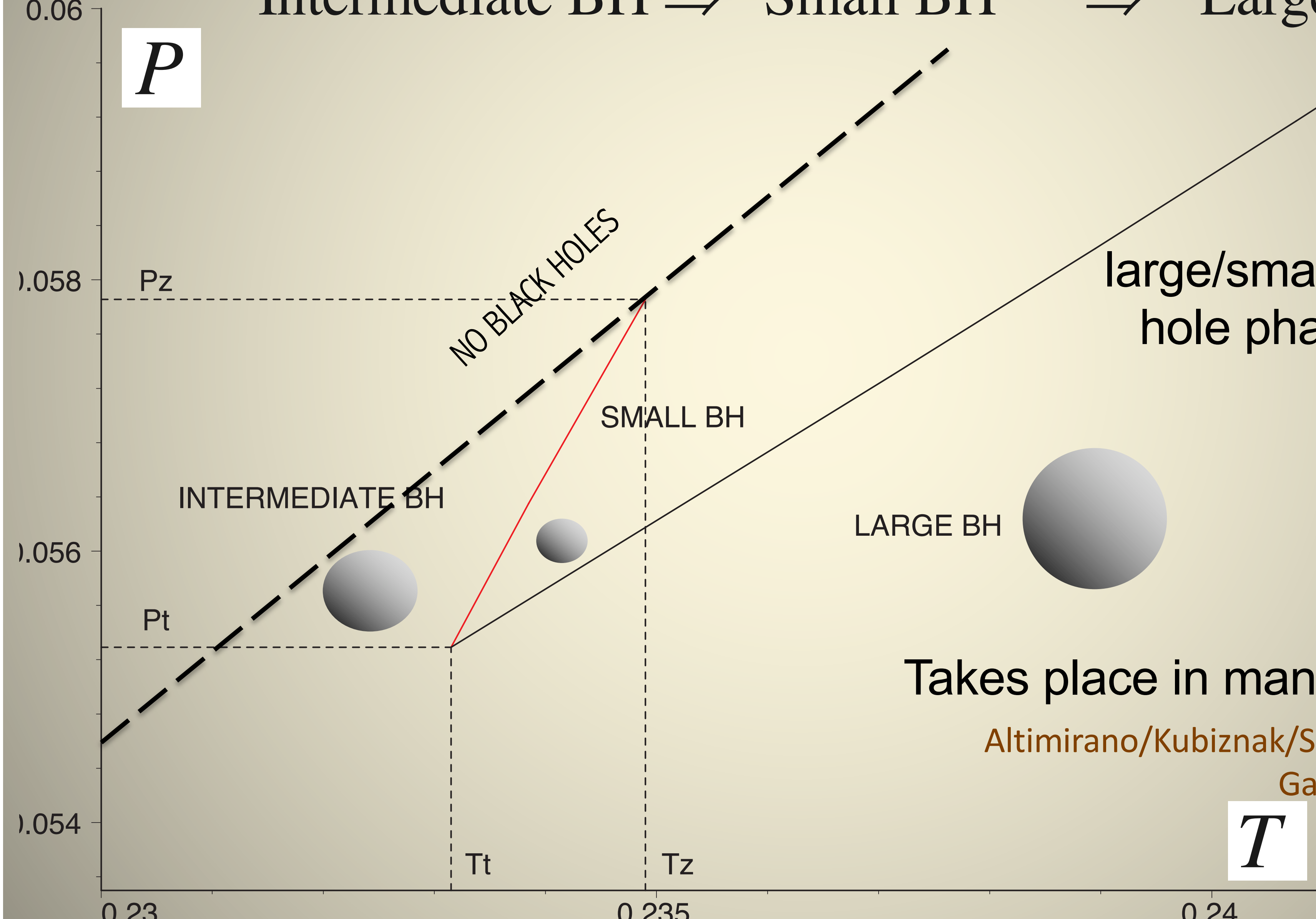
Reentrant Phase Transitions in D>5



Low T | **Medium T** | **High T**

mixed $\Rightarrow$ water/nicotine $\Rightarrow$ mixed

Intermediate BH $\Rightarrow$ Small BH $\Rightarrow$ Large BH



large/small/large black hole phase transition

Takes place in many examples

Altimirano/Kubiznak/Sherkatgnad/Mann
Galaxies **2** (2014) 89

Multiply Rotating Kerr-AdS Black Holes

$$ds^2 = -W\Big(1+\frac{r^2}{l^2}\Big)d\tau^2 + \frac{2m}{U}\Big(W d\tau - \sum_{i=1}^{N}\frac{a_i\mu_i^2 d\varphi_i}{\Xi_i}\Big)^2$$

Kerr-AdS

$$+\sum_{i=1}^{N}\frac{r^2+a_i^2}{\Xi_i}\mu_i^2 d\varphi_i^2 + \frac{U dr^2}{F-2m} + \sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}d\mu_i^2$$

$$-\frac{l^{-2}}{W(1+r^2/l^2)}\Big(\sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}\mu_i d\mu_i\Big)^2$$

$$n = \frac{1}{2}(d-1-\epsilon)$$

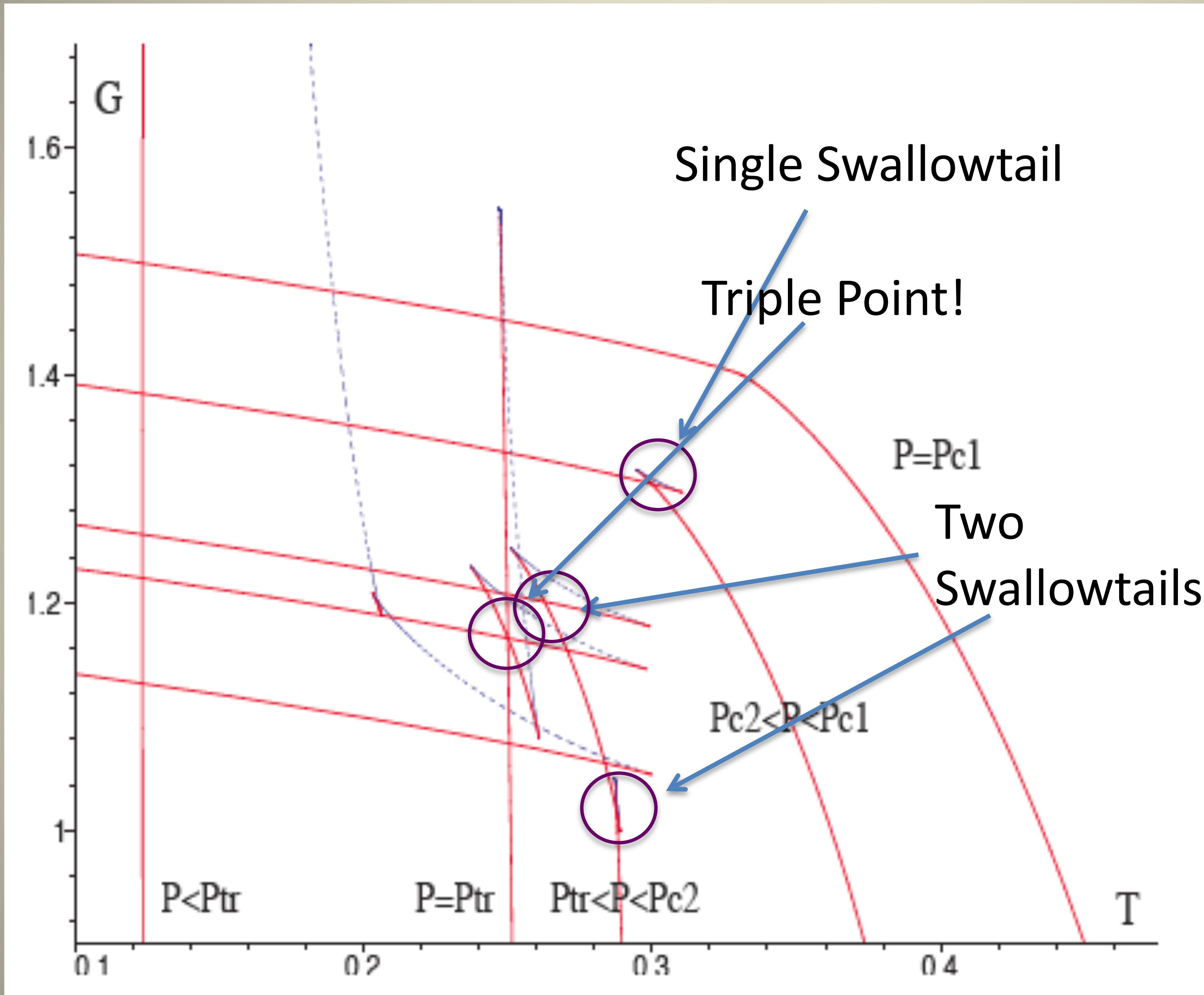
$$\epsilon = 0/1\,\text{odd/even}\,D$$

$$W = \sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{\Xi_i} \qquad U = r^\epsilon\sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{r^2+a_i^2}\prod_j (r^2+a_j^2)$$

$$\sum_{i=1}^{n}\mu_i^2 = 1$$

$$F = r^{\epsilon-2}\Big(1+\frac{r^2}{l^2}\Big)\prod_{i=1}^{N}(r^2+a_i^2) \qquad \Xi_i = 1-\frac{a_i^2}{l^2}$$

Triple Points: Multiply Rotating Black Holes



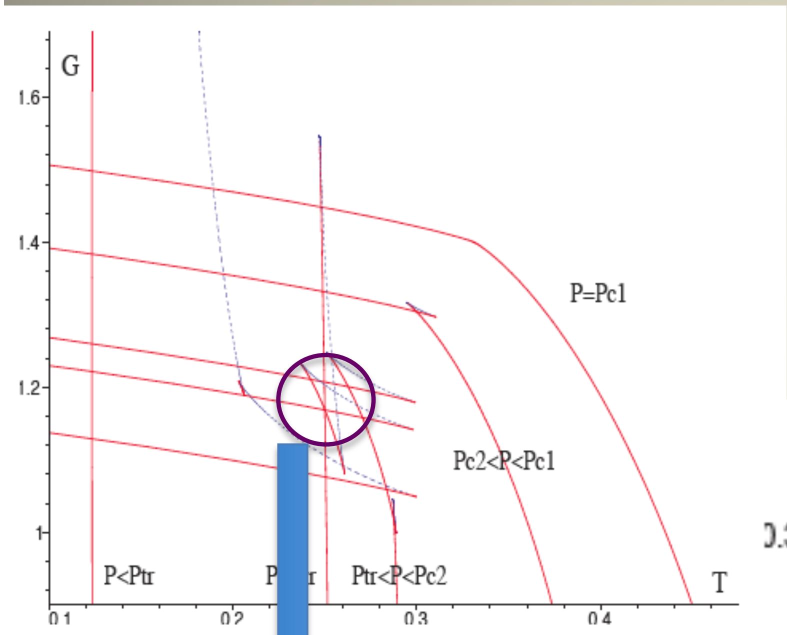
$$\frac{J_2}{J_1} = 0.05$$

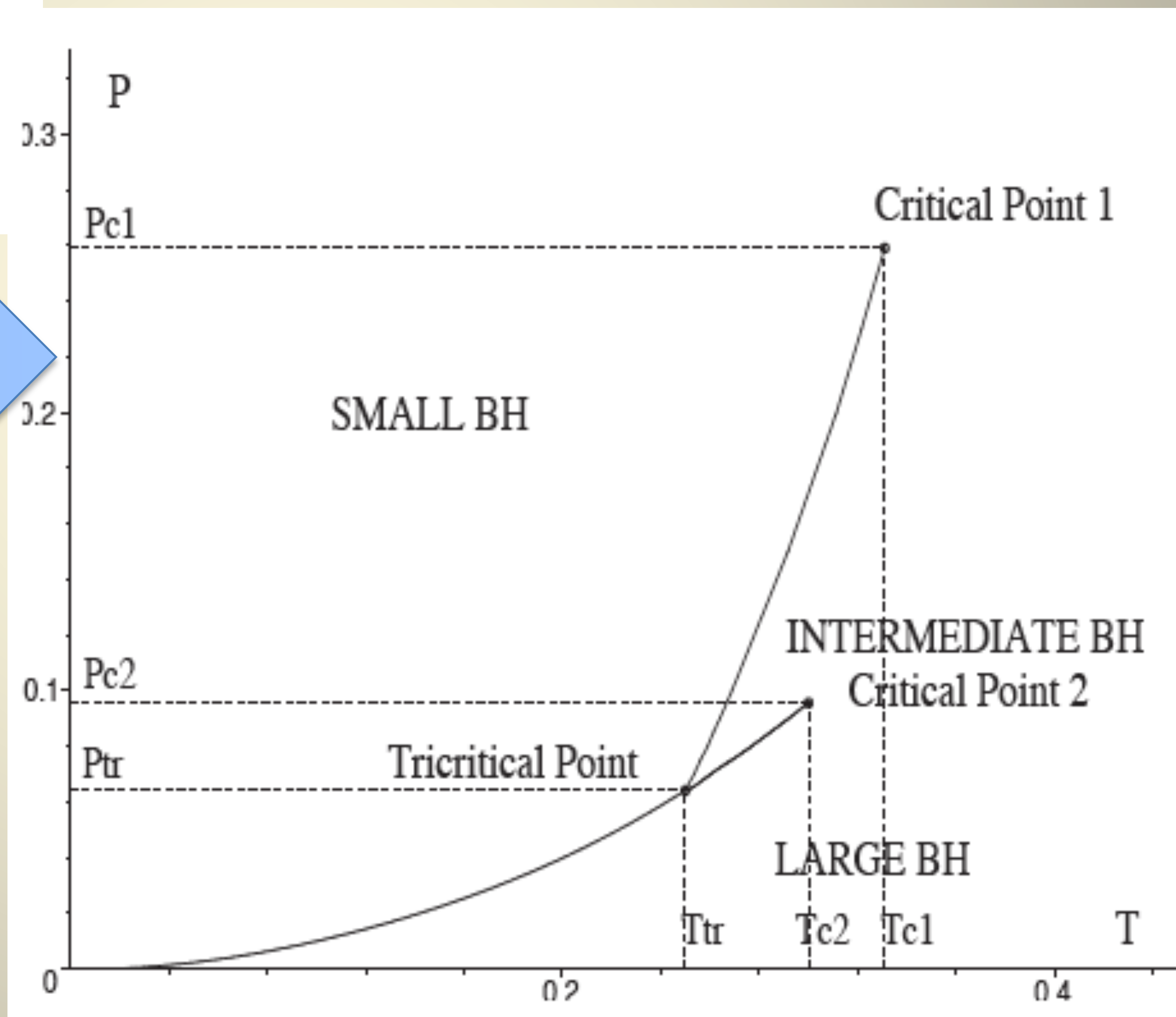
$$P_{c1} = 0.259$$

$$P_{c1} = 0.0956$$

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
CQG 31 (2104) 042001 arXiV:1308.2672

Reentrant Phase transition Ending in a Triple Point

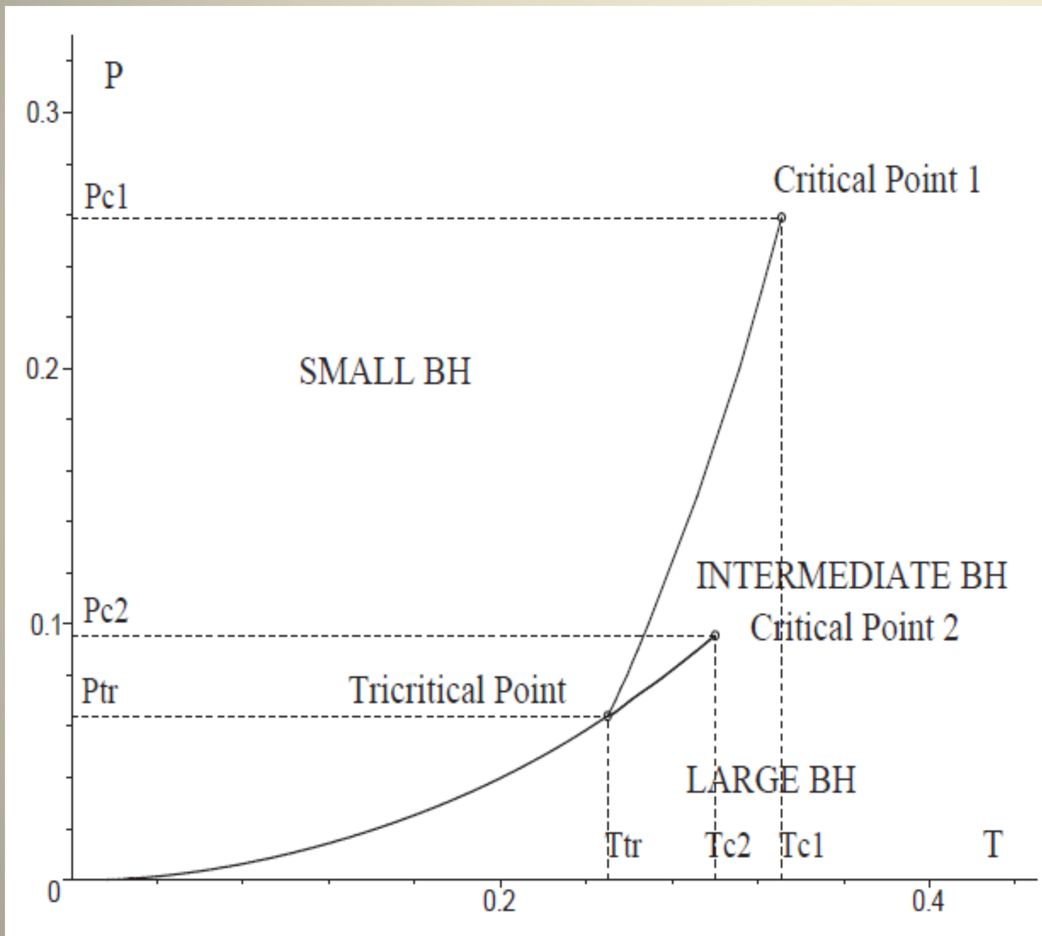


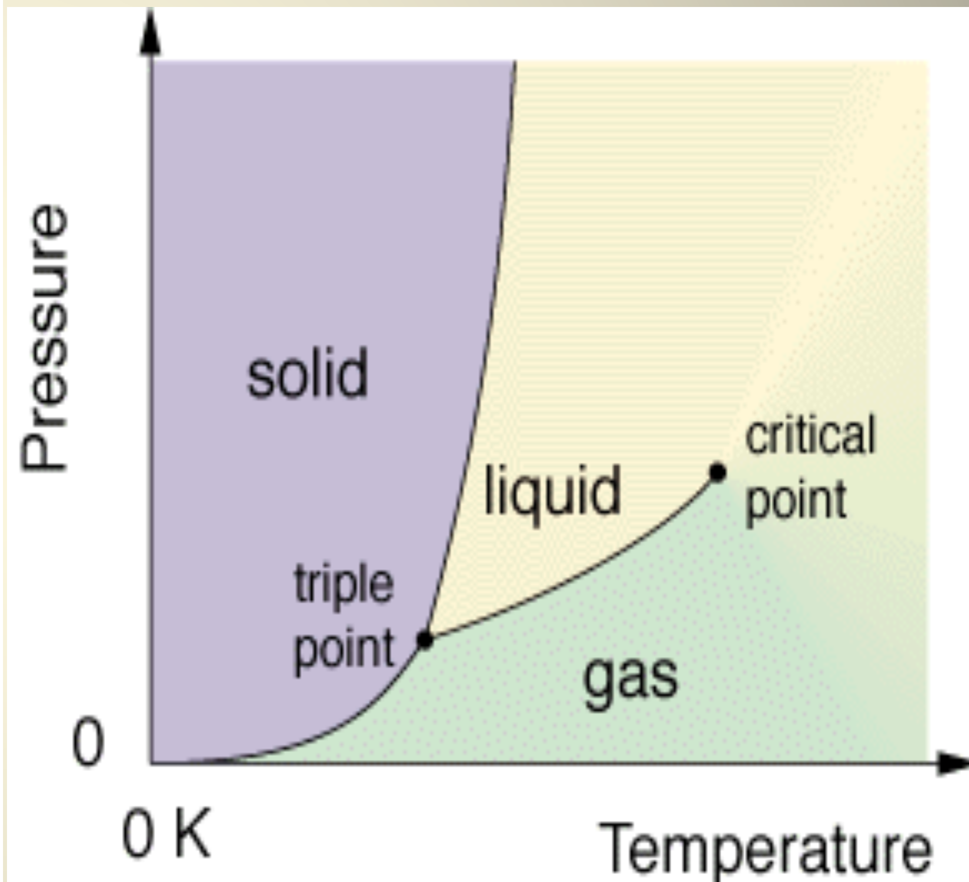


$$\frac{J_2}{J_1} = 0.05$$

The Black Hole Triple Point

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, RBM CQG **31** (2014) 042001





Also for charged black holes in Einstein-Gauss Bonnet Gravity!

S. Wei, Y. Liu PRD **90** (2014) 044057