BLACK HOLE CHEMISTRY

Robert Mann

University of Waterloo
Waterloo, Canada

https://boardgamegeek.com/image/2241156/alchemists

Recent Reviews

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. **00** (2017) 000000 (66pp) UNCORRECTED PROOF

Topical Review

Black hole chemistry: thermodynamics with Lambda

David Kubizňák[1,2], Robert B Mann[1,2] and Mae Teo[3]

[1] Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
[2] Perimeter Institute for Theoretical Physics, 31 Caroline St. N.,Waterloo, Ontario N2L 2Y5, Canada
[3] Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA 94305, United States of America

E-mail: dkubiznak@perimeterinstitute.ca, rbmann@uwaterloo.ca and maehwee@stanford.edu

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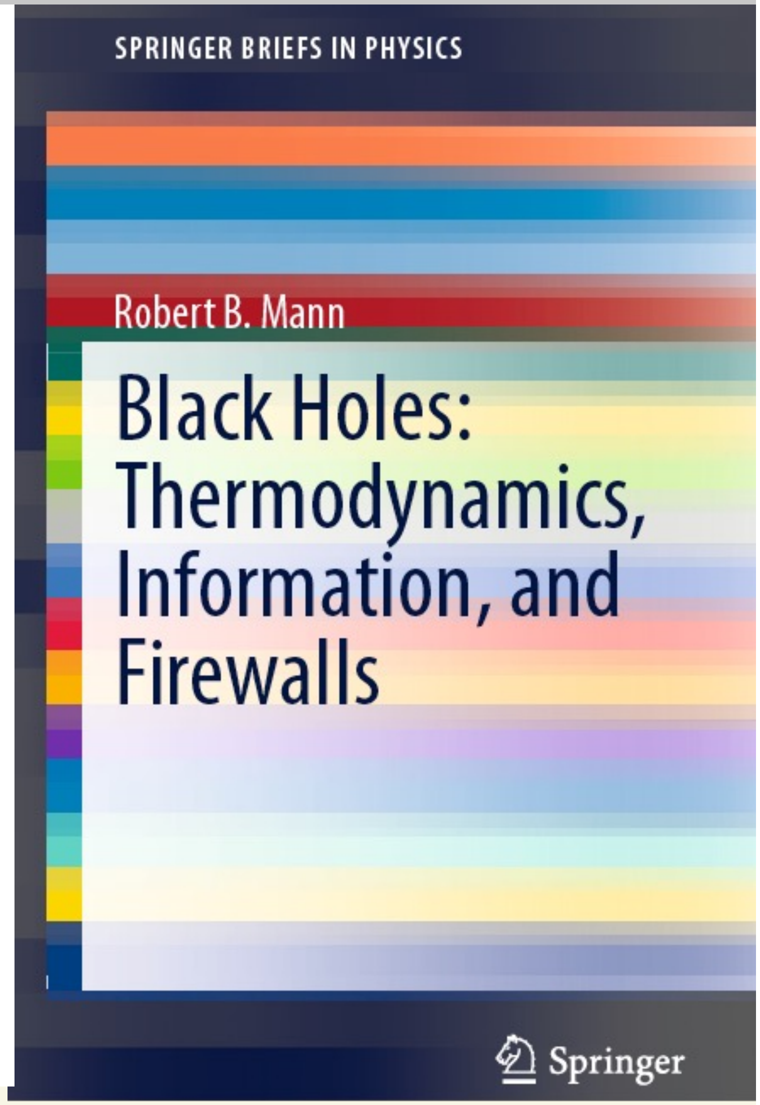
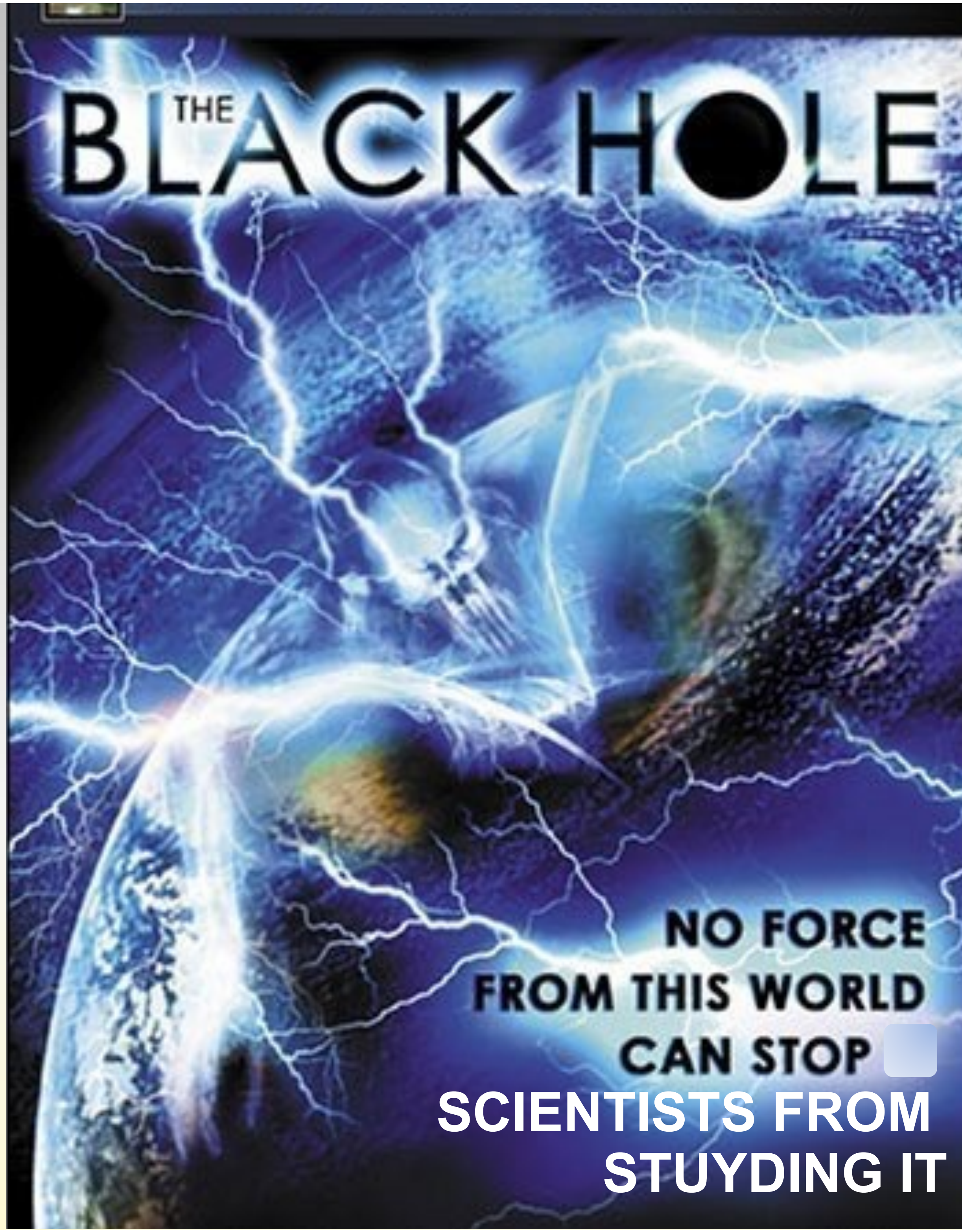
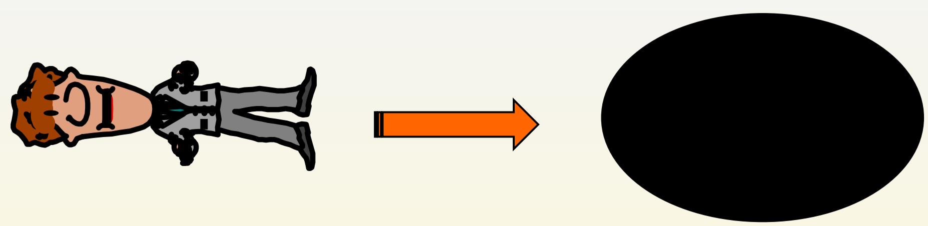



THE
BLACK HOLE
NO FORCE
FROM THIS WORLD
CAN STOP
SCIENTISTS FROM
STUYDING IT

Dark Stars → Black Holes

If the semi-diameter of a sphaere of the same density with the sun were to exceed that of the sun in the proportion 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

Rev. John Michell (1724-1793)



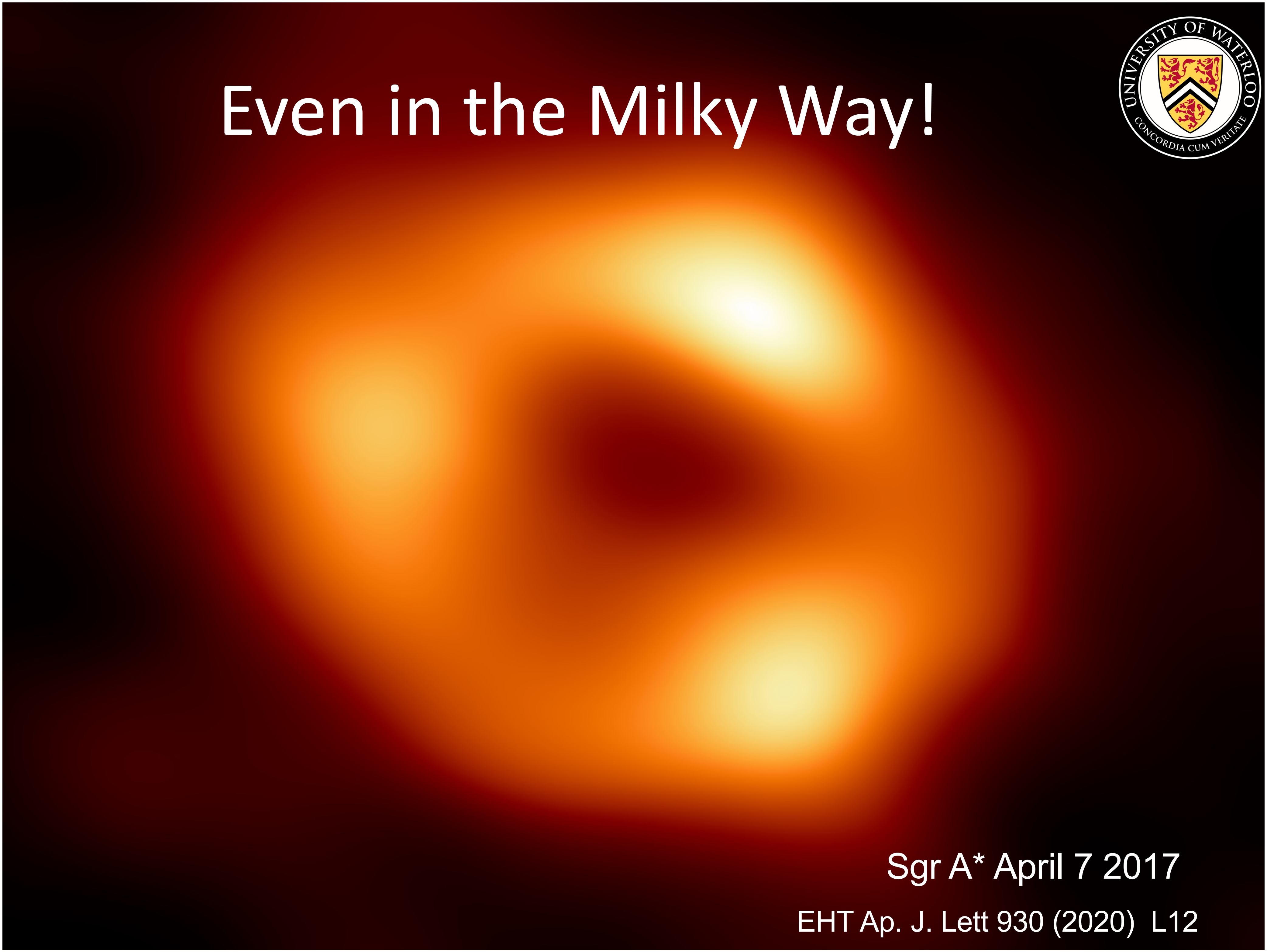
Dark Star (1783): A region of space for which the escape velocity is greater than the speed of light

Black Holes Exist!

First M87 Event Horizon Telescope Results

EHT Ap. J. Lett 875 (2019) L1

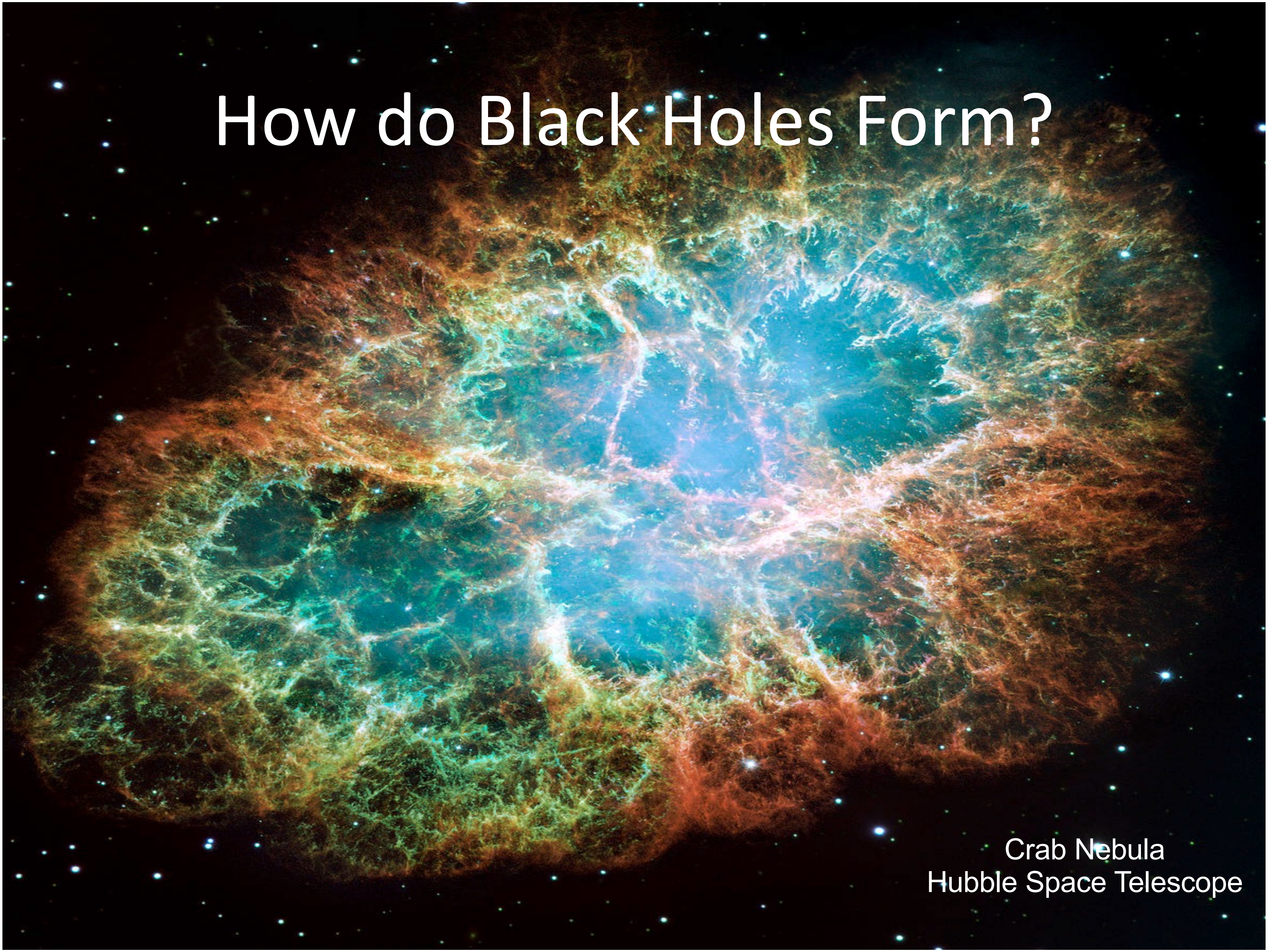
Even in the Milky Way!



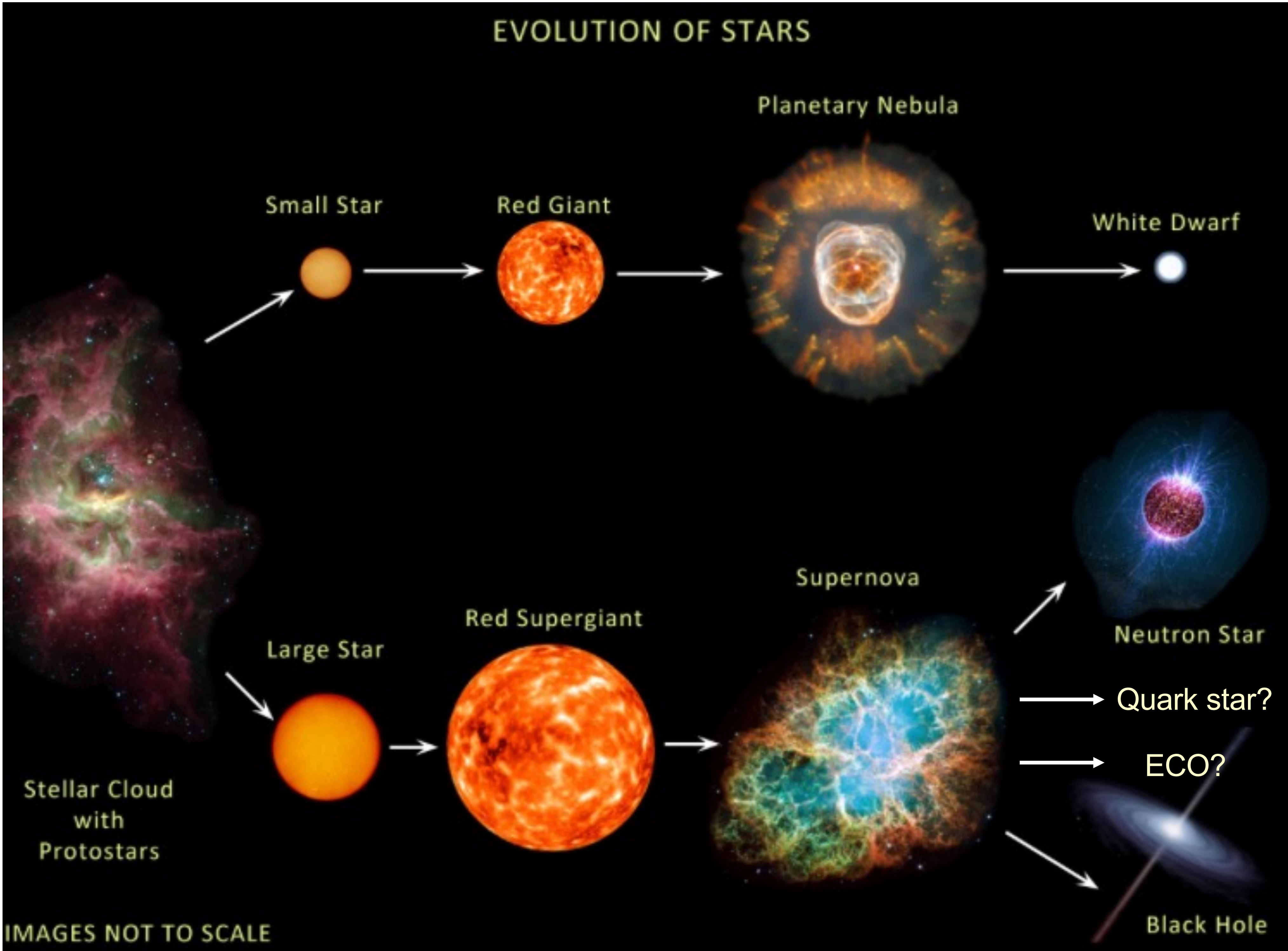
Sgr A* April 7 2017

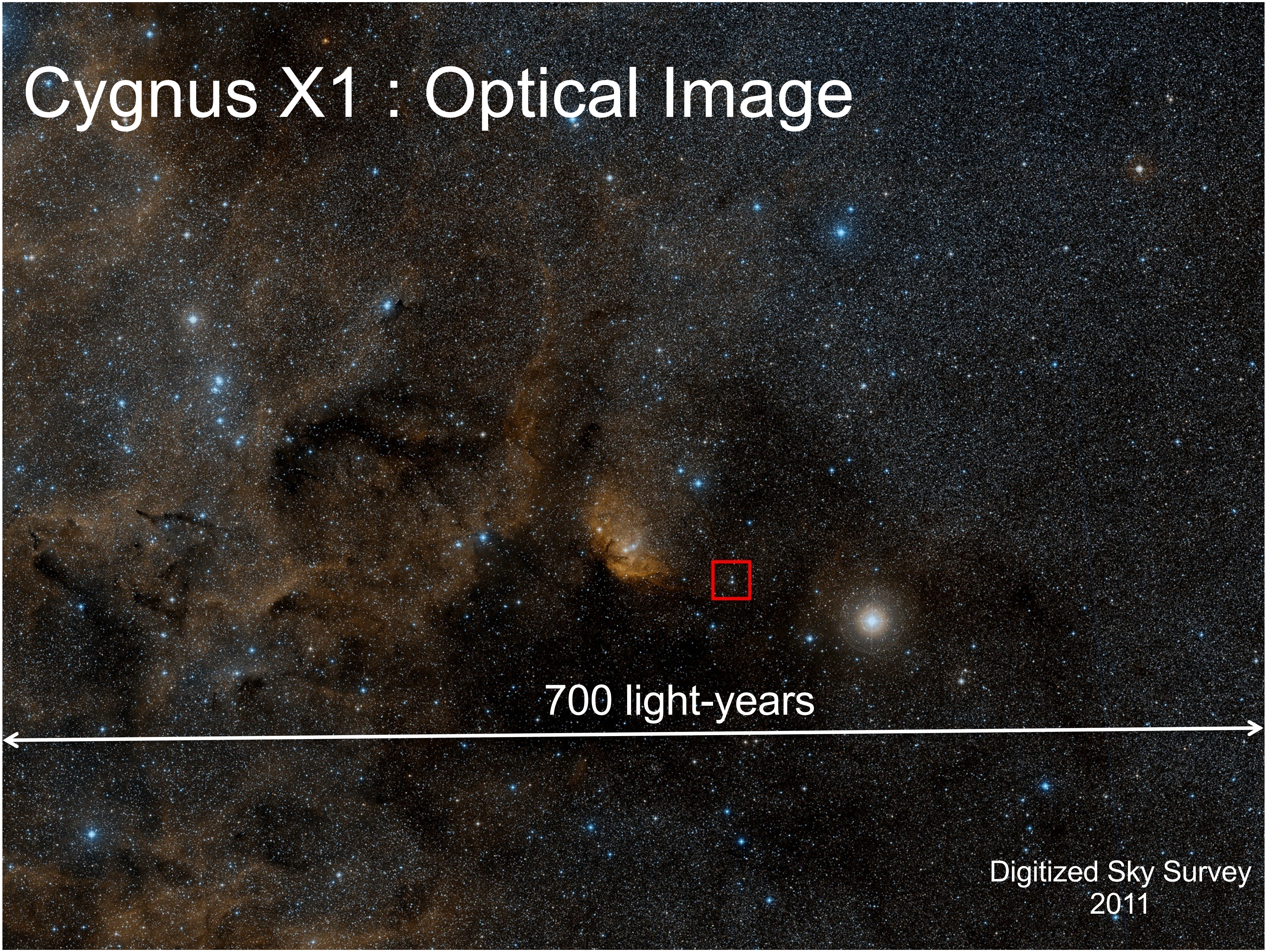
EHT Ap. J. Lett 930 (2020) L12

How do Black Holes Form?

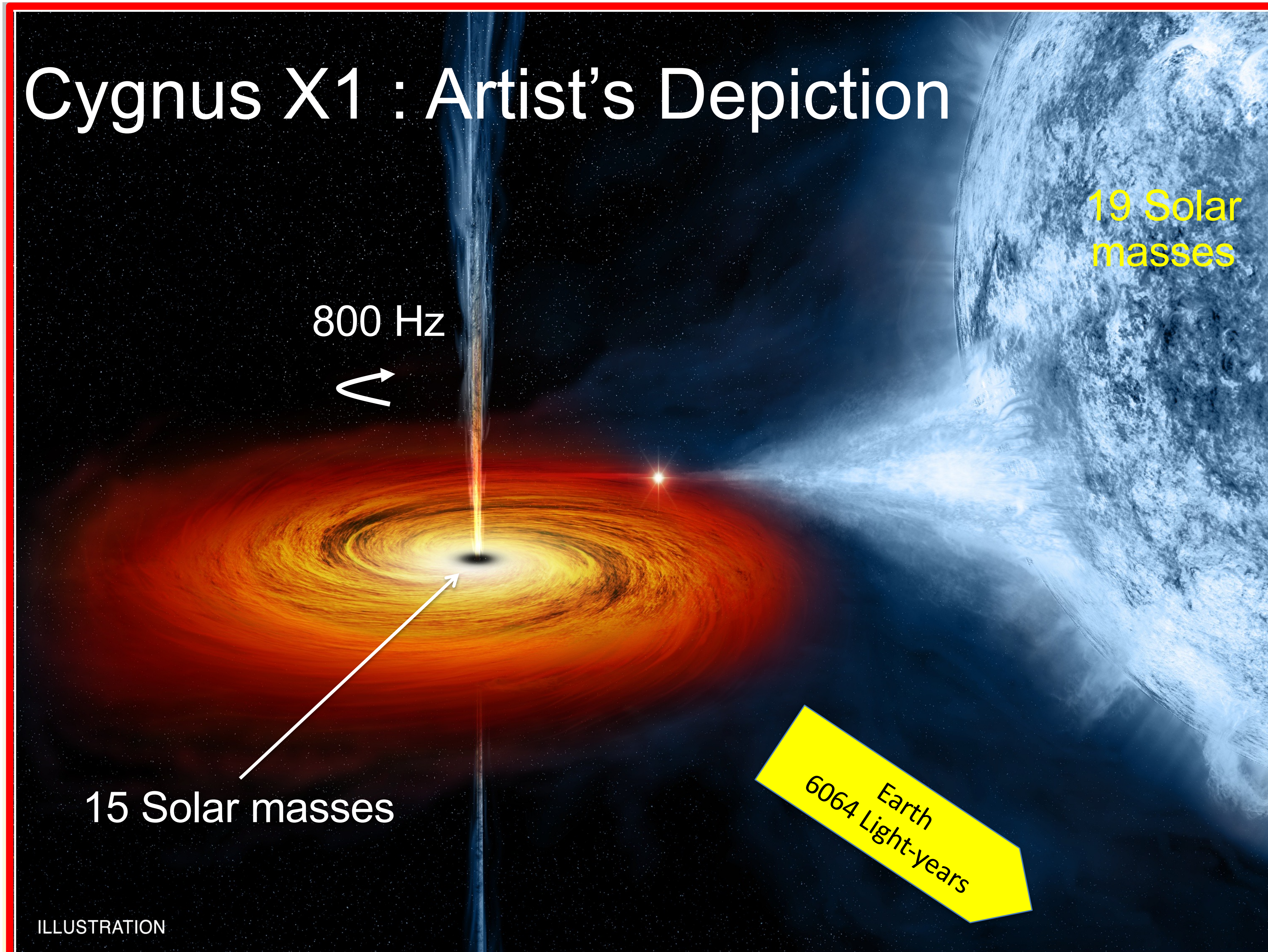


Crab Nebula
Hubble Space Telescope


EVOLUTION OF STARS
Small Star
Red Giant
Planetary Nebula
White Dwarf
Stellar Cloud
with
Protostars
Large Star
Red Supergiant
Supernova
Neutron Star
Quark star?
ECO?
Black Hole
IMAGES NOT TO SCALE


Cygnus X1 : Optical Image
700 light-years
Digitized Sky Survey
2011

Cygnus X1 : Artist's Depiction

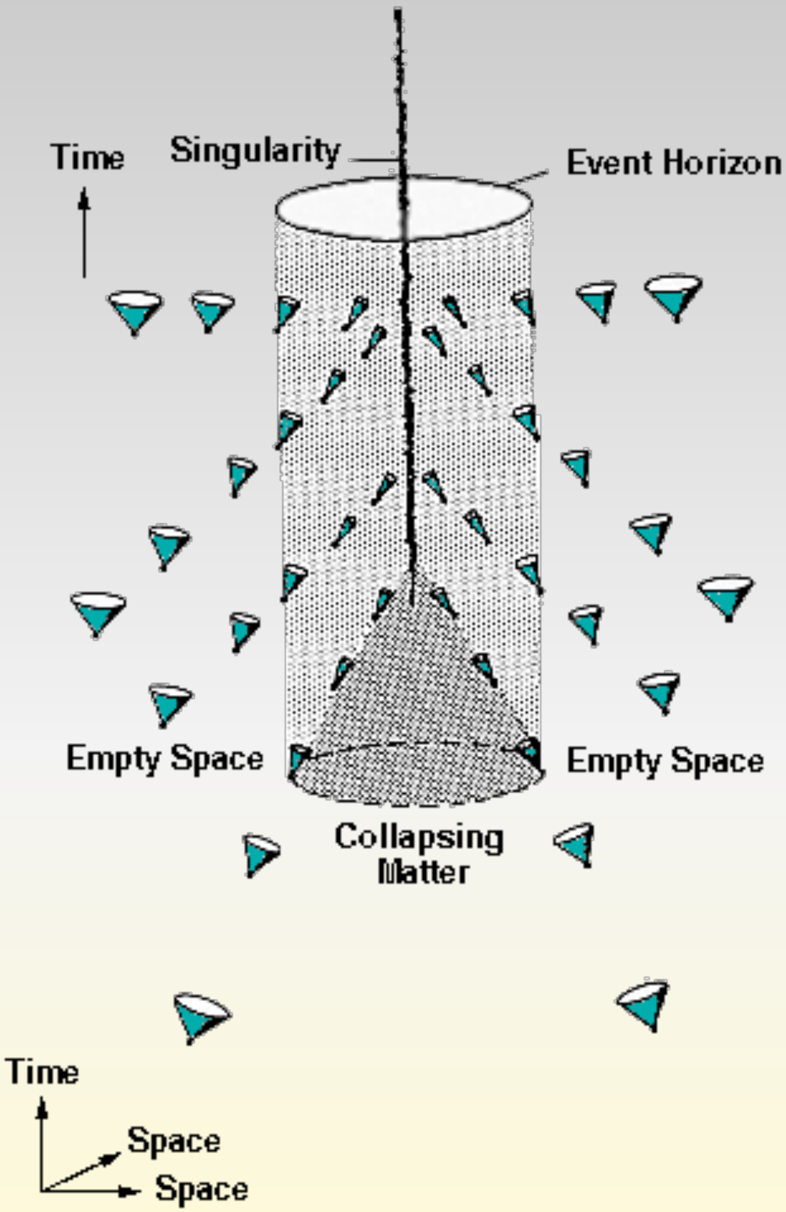


Astrophysical Black Hole Properties

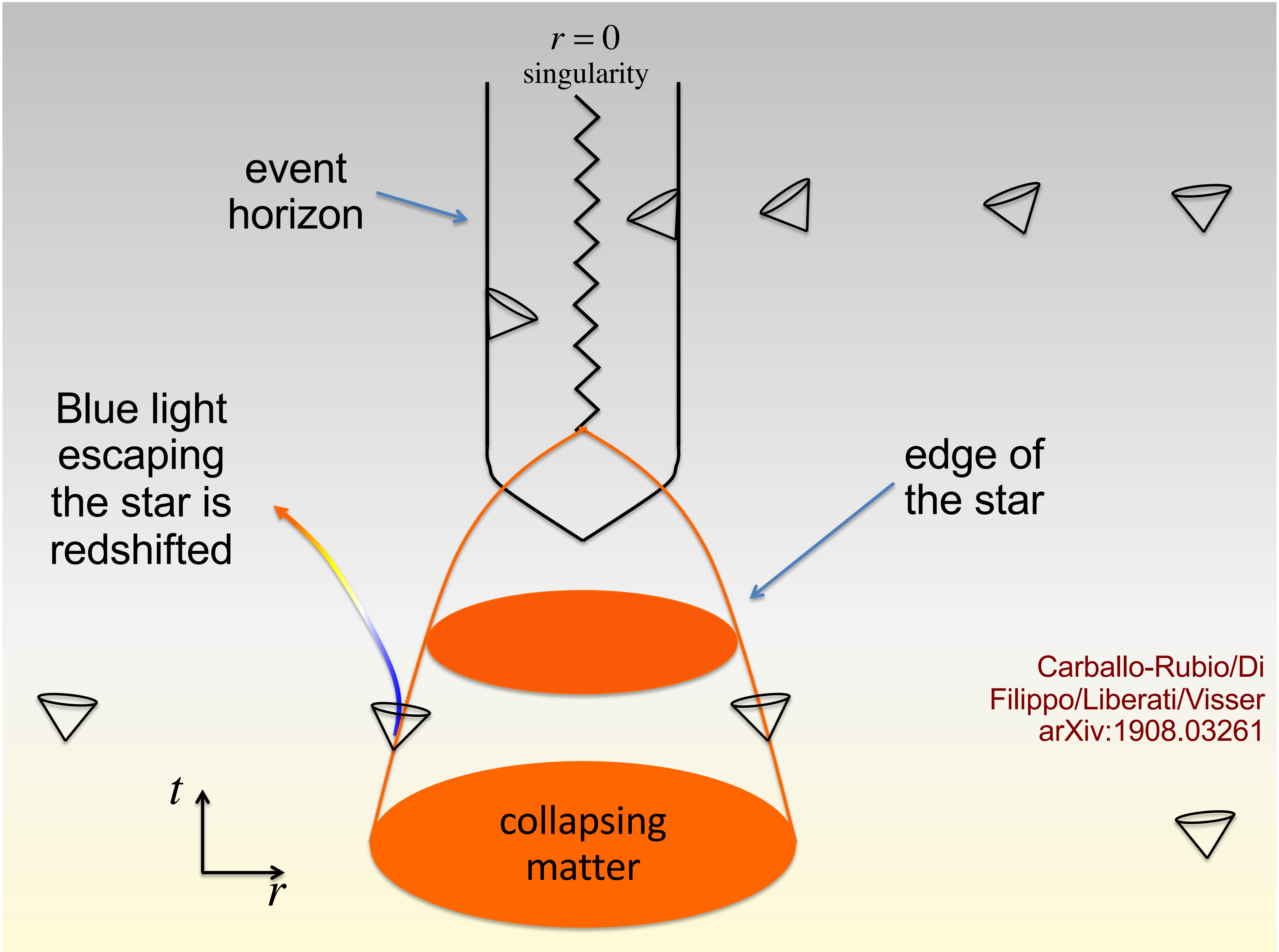
Class	Approx. mass	Approx. radius
Supermassive black hole	10^5–10^{10} $M_\odot$	0.001–400 AU
Intermediate-mass black hole	10^3 $M_\odot$	10^3 km ≈ R_{Earth}
Stellar black hole	10 $M_\odot$	30 km
Micro black hole	up to M_{Moon}	up to 0.1 mm

- Spins up to near extremal
- Magnetic Fields
- Accretion Disks, Jets
- Gravitational Waves
- Do they accelerate?

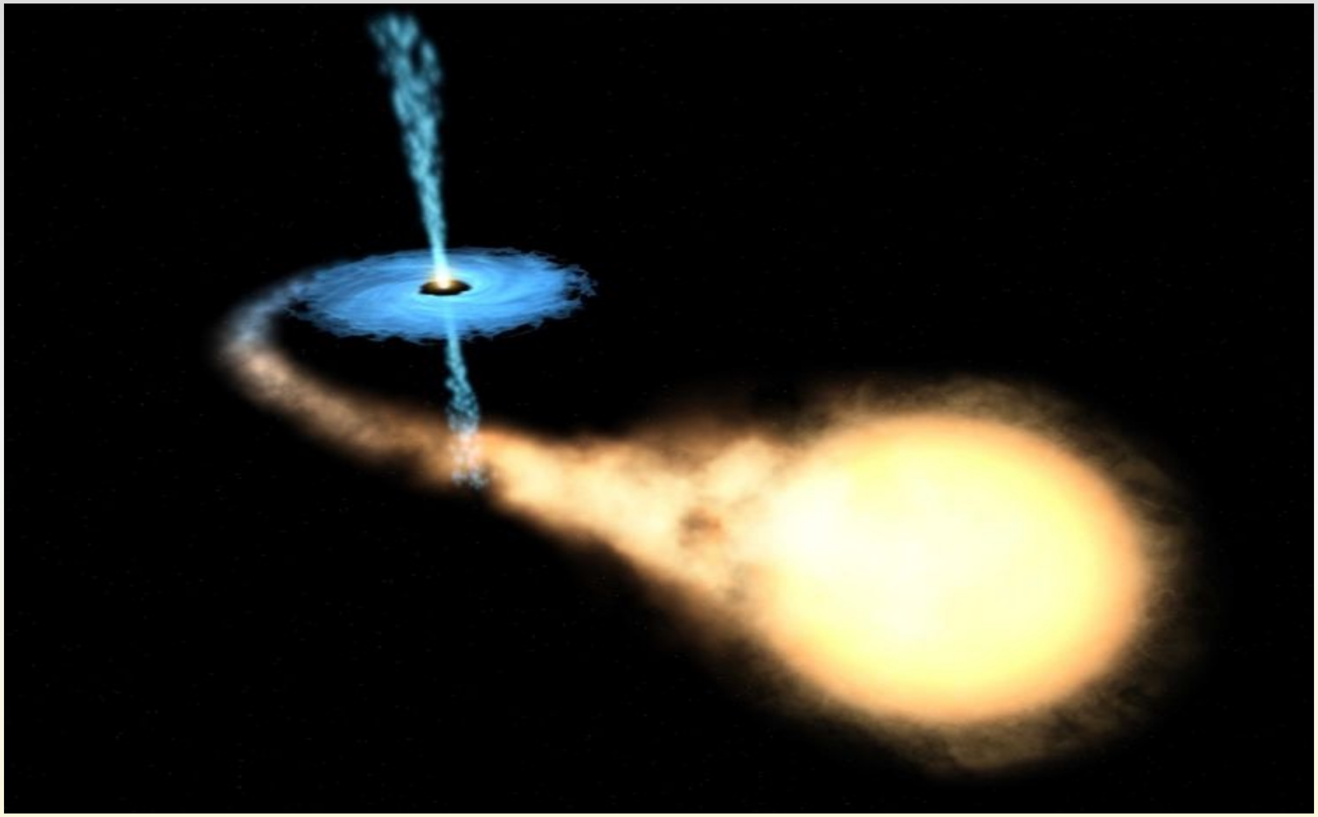
The Strange Properties of Black Holes



- A singularity at the core where time and space no longer exist
- Inevitable result of gravitational collapse (gravity always wins!)
- Can be mined for energy (if they spin)
- Collide and produce gravity waves
- Behave as thermodynamic objects
- Can be produced in pairs in the early universe
- Source of quantum information paradoxes (eg firewalls)

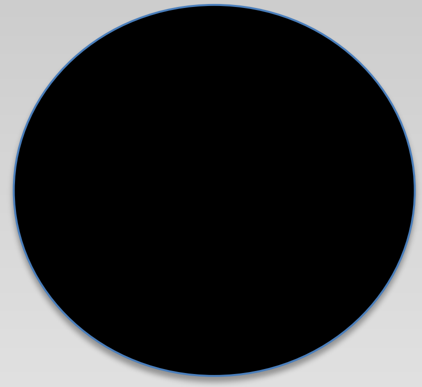

r = 0
singularity
event
horizon
Blue light
escaping
the star is
redshifted
edge of
the star
Carballo-Rubio/Di
Filippo/Liberati/Visser
arXiv:1908.03261
collapsing
matter
t
r

Black Holes as Thermodynamic Objects



Area Law

What happens if you pour a cup of tea into a black hole?



- Tea is hot – has entropy
- Black hole absorbs everything and has no structure
- Where does the entropy go?

Bekenstein: Tea has mass → will increase mass of black hole
→ area must increase

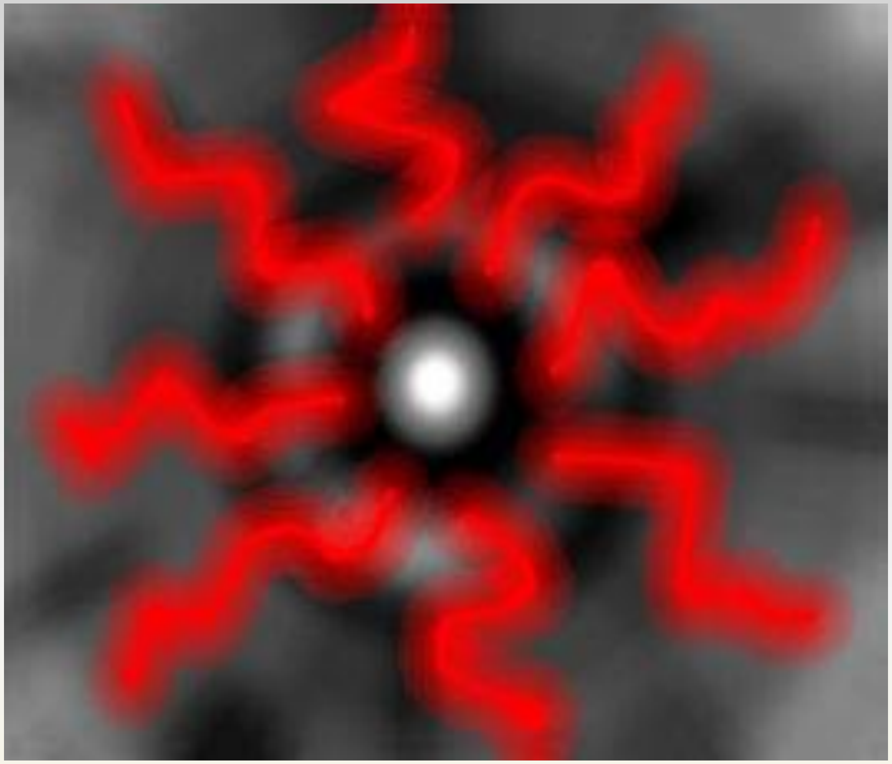
Bekenstein PRD **7** (1973) 2333

A(area) ⟺ S (entropy)

Hawking
Nature 248 (1974) 30ikh

Black Hole Temperature

When quantum effects are taken into account, black holes radiate away particles as black body.



$$T = \frac{\kappa}{2\pi}\,, \qquad S = \frac{A}{4}$$

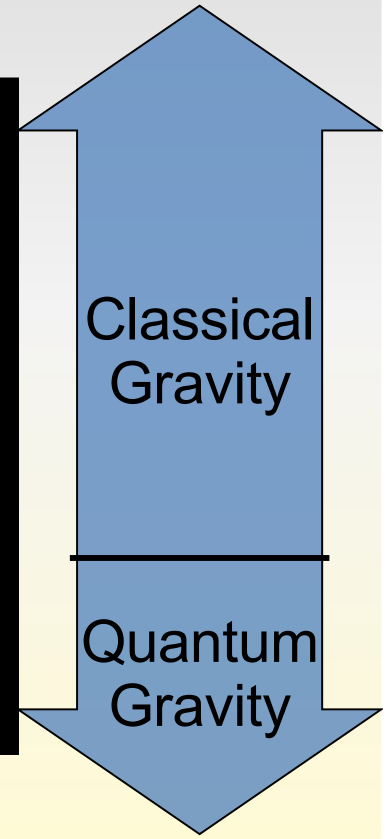
Five approaches:

- QFT Bogoliubov (original)
- Euclidean path integrals
- Tunneling
- Require quantum state of BH exterior to solve semi-classical Einstein equations
- Prevent anomalous breaking of diffeomorphism symmetry

Hot Black Holes?

- Semiclassical quantum physics indicates that black holes behave like hot objects that are maximally disordered (they have maximal entropy)
- Temperature increases with decreasing mass
- As the hole radiates, it gets hotter!
- Nobody knows what the end-point of this process is

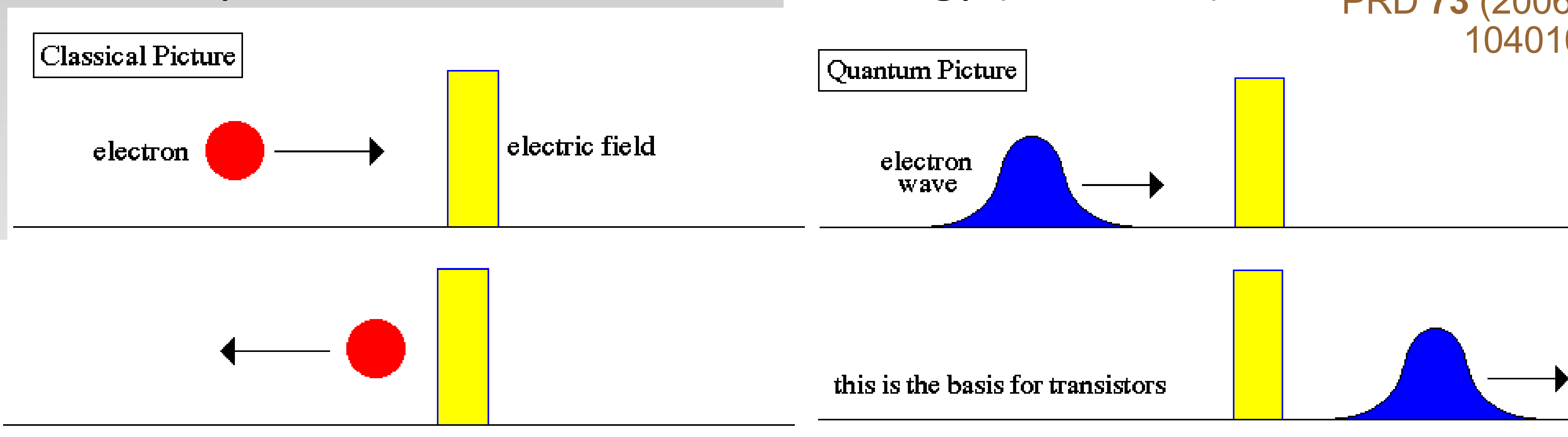
Milky Way BH: $T = 1.4 \times 10^{-14}\ {}^{\mathrm{o}}K$	$R = 1.27 \times 10^{6}$ km
Sun: $T = 6.2 \times 10^{-8}\ {}^{\mathrm{o}}K$	$R = 2.95$ km
Mercury : $T = 0.373\ {}^{\mathrm{o}}K$	$R = .049$ cm
Varuna: $T = 331\ {}^{\mathrm{o}}K$	$R = 0.550\ \mu$m
Mt. Everest: $T = 4.3 \times 10^{8}\ {}^{\mathrm{o}}K$	$R = 4.5 \times 10^{-12}$ m
Proton: $T = 7.3 \times 10^{49}\ {}^{\mathrm{o}}K$	$R = 2.5 \times 10^{-54}$ m



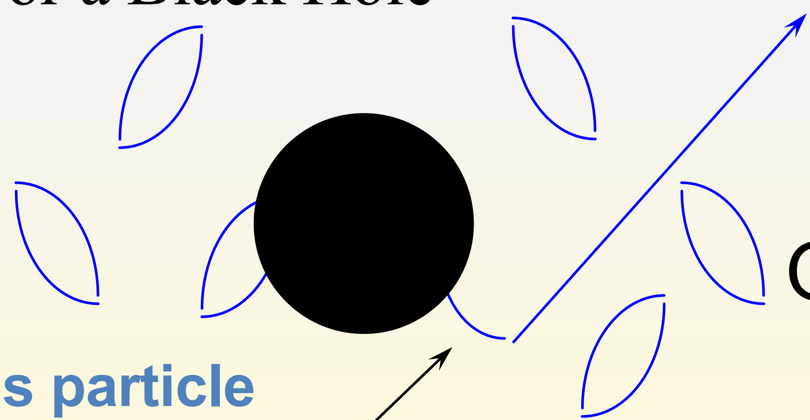
Quantum Black Holes

- Quantum effects permit particles to tunnel out of the gravitational potential well
- as they do so, the black hole loses energy (and mass)

Parikh/Wilczek PRL **85** (2000) 5042
Vanzo et.al. JHEP **0505** (2005) 014
Kerner/Mann PRD **73** (2006) 104010



For a Black Hole



this particle escapes out to infinity causing the black hole to lose energy

this particle falls in the hole

Quantum particles tunnel out of the horizon

Kerner/Mann CQG **25** (2008) 095014

A black hole is a hot particle radiator!

Information Paradox

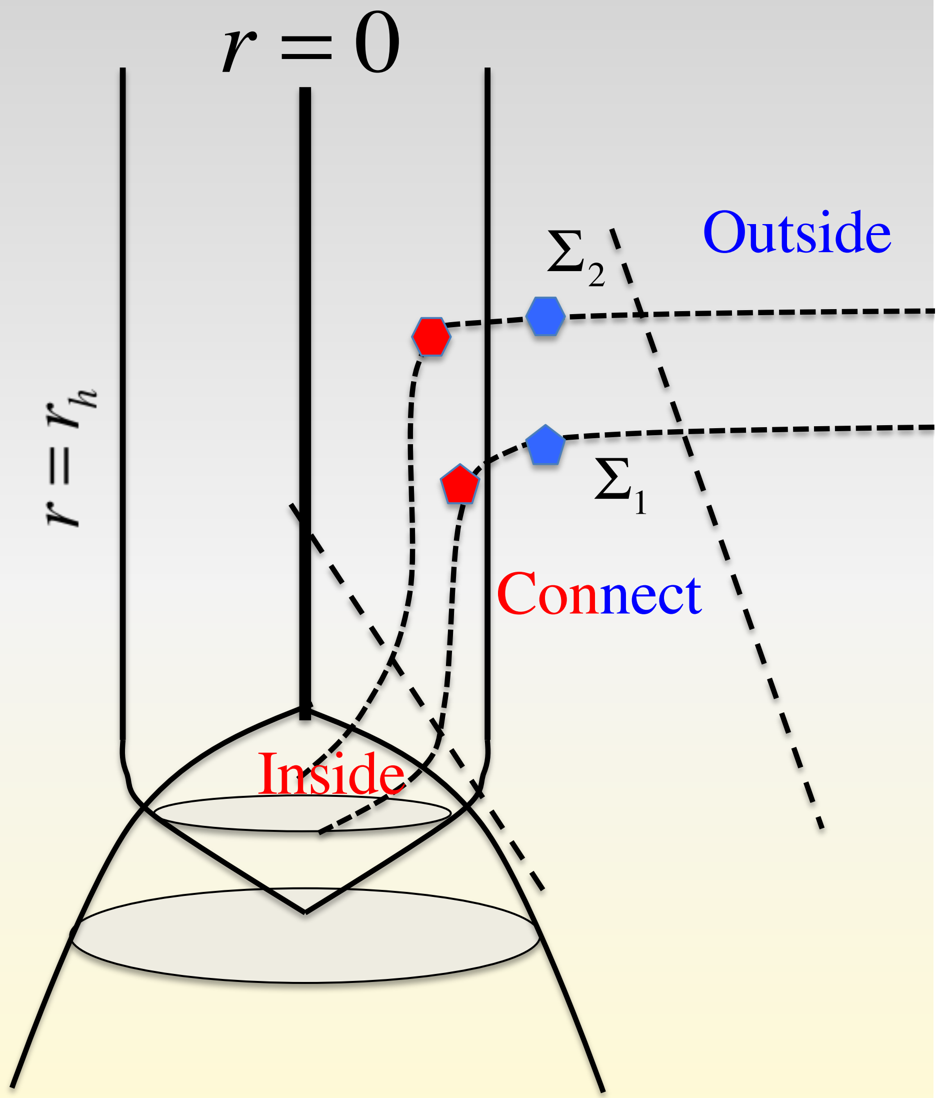
Hawking PR**D14** (1976) 2460
S. Mathur CQG **26** (2009) 224001

Outside: t=constant
Inside: r=constant

$$|\Psi\rangle_1 \simeq \frac{1}{\sqrt{2}}|\Phi\rangle_{I1}\otimes\left(|0_k\rangle_{I1}|0_{-k}\rangle_{O1}+|1_k\rangle_{I1}|1_{-k}\rangle_{O1}\right)$$

$$\rho_{O_1} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{ent}(1) = -\mathrm{Tr}\left[\rho_{O_1}\log\rho_{O_1}\right]$$
$$= 2\times\frac{1}{2}\log 2 = \log 2$$



Next emission

$$|\Psi\rangle_2 \approx \frac{1}{2}|\Phi\rangle_{I2} \otimes \left(|0_k\rangle_{I1}|0_{-k}\rangle_{O1} + |1_k\rangle_{I1}|1_{-k}\rangle_{O1}\right) \otimes \left(|0_k\rangle_{I2}|0_{-k}\rangle_{O2} + |1_k\rangle_{I2}|1_{-k}\rangle_{O2}\right)$$

$$\rho_{O_1} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \mathrm{diag}\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right) \quad S_{ent}(2) = -\mathrm{Tr}\left[\rho_{O2}\ln\rho_{O2}\right] = 2\ln 2$$

*n*th emission

$$|\Psi\rangle_n \approx \frac{1}{2^{n/2}}|\Phi\rangle_{In} \prod_{m=1}^{n} \otimes \left(|0_k\rangle_{Im}|0_{-k}\rangle_{Om} + |1_k\rangle_{Im}|1_{-k}\rangle_{Om}\right)$$

$$\rho_{O_n} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \mathrm{diag}\left(2^{-n},2^{-n},\ldots,2^{-n}\right)$$

$$S_{ent}(n) = -\mathrm{Tr}\left[\rho_{On}\ln\rho_{On}\right] = n\ln 2$$

Entropy grows unboundedly!

$$n \rightarrow \infty$$

$$n = \sigma^{-1}\left(\frac{M}{M_p}\right)^2$$

$M = M_{\odot}$

$$n \simeq 10^{76}$$

Solutions?

- Remnants
  - Something terminates evolution once $M = M_r \geq M_{Pl}$
  - Remnant must be *n*-fold degenerate since its entanglement with radiation is *n log2*
  - Each remnant state gives finite loop correction to scattering processes → sum over *n* is divergent unless its couplings vanish
- Mixedness
  - Black hole evaporates leaving radiation with entanglement entropy *n log2* but unentangled with any quantum state
  - Initial pure state evolves to mixed state → violates unitarity
- Bleaching
  - Information can never enter the black hole
  - Some strange process decouples the information of a state from its energy and momentum
  - Initial state should never have formed the black hole in the first place

Black Hole Thermodynamics

- Basic idea: Black holes are thermodynamic systems
- Stationary black holes ←→ Equilibrium states
- Other black holes: non-equilibrium states
- Conserved quantities: no-hair theorem
  - M, Q, J (multiple Q's and J's possible in higher dim)

The Laws of Thermodynamics

- 0^{th}: Temperature is constant at equilibrium
  - 2 systems, each in thermal equilibrium with a 3^{rd}, are in thermal equilibrium with each other
- 1^{st}: Energy is conserved $\Delta U = Q - W$
  - Change in energy = heat supplied – work done
- 2^{nd}: Entropy never adiabatically decreases
  - Adiabatic: operation taking system from one equilibrium state to another without coupling it to other thermodynamic systems (no heat flow)

$$S = S(U, X_1, \ldots, X_n) \qquad \Delta S \geq 0$$

The 4 Laws of Black Hole Mechanics

Bardeen/Carter/Hawking CMP **31** (1973) 161

- 0th Law $\kappa = \text{constant}$
  - surface Gravity is constant over the event horizon
- 1st Law $dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ + \cdots$
  - differences in mass between nearby solutions are equal to differences in area times the surface gravity plus additional work terms
- 2nd Law $dA \geq 0$

  Bekenstein PRD **7** (1973) 2333
  - area of the event horizon never decreases in any physical process
- 3rd law $\kappa_n > \kappa_{n+1} > 0 \quad n < \infty$

  Israel PRL **57** (1986) 397
  - No procedure can reduce the surface gravity to 0 in a finite number of steps

Black Hole Thermodynamics

Thermodynamics | Gravity

Energy $E \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \dfrac{\hbar\kappa}{2\pi}$ Surface gravity

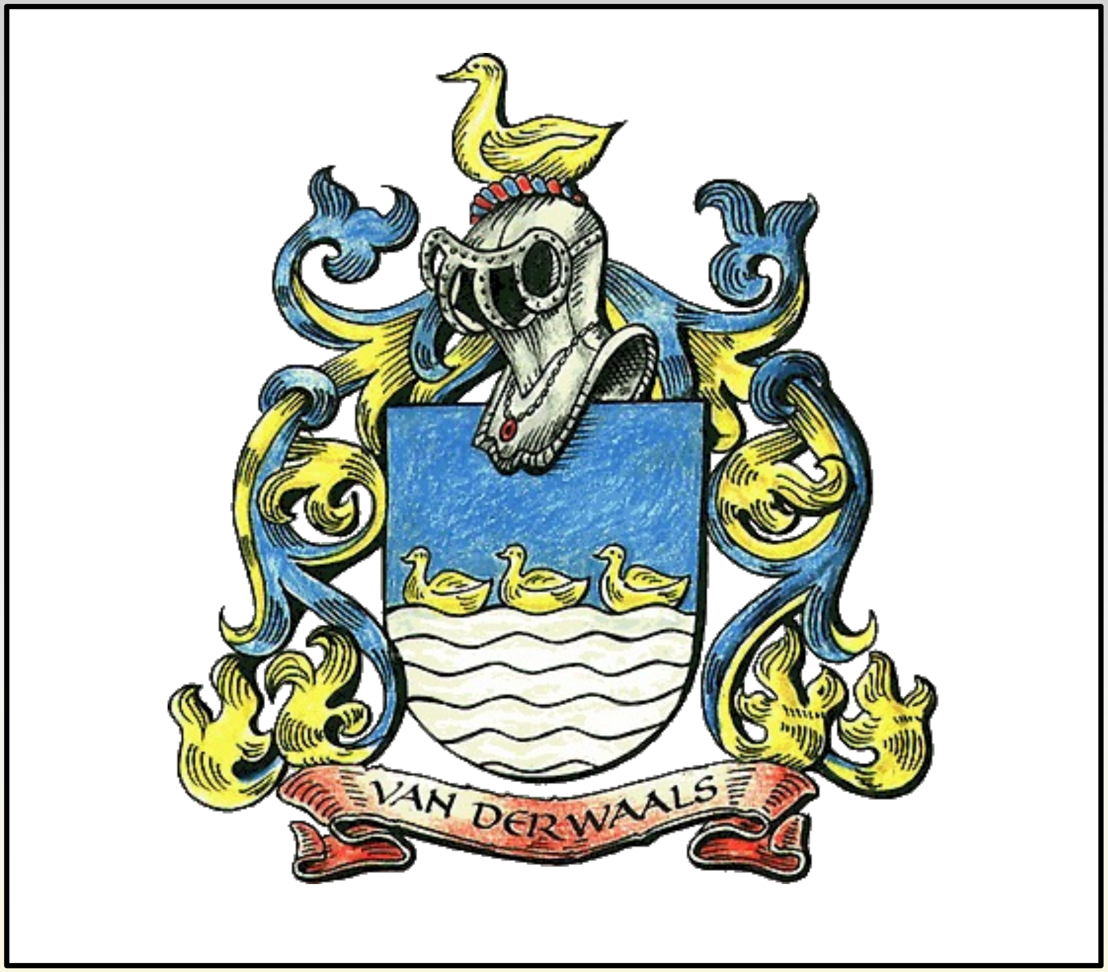
Entropy $S \leftrightarrow \dfrac{A}{4\hbar}$ Horizon Area

$$dE = TdS + VdP + \text{work terms} \leftrightarrow dM = \frac{\kappa}{8\pi}dA + \Omega dJ + \Phi dQ$$

First Law | First Law

?

Black Hole Chemistry



Johannes Diderik van der Waals (1837-1923)

Cosmological constant: Einstein's biggest blunder?
In order to obtain static Universe Einstein introduced:

$$G_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$$

Curvature of spacetime

Vacuum Energy

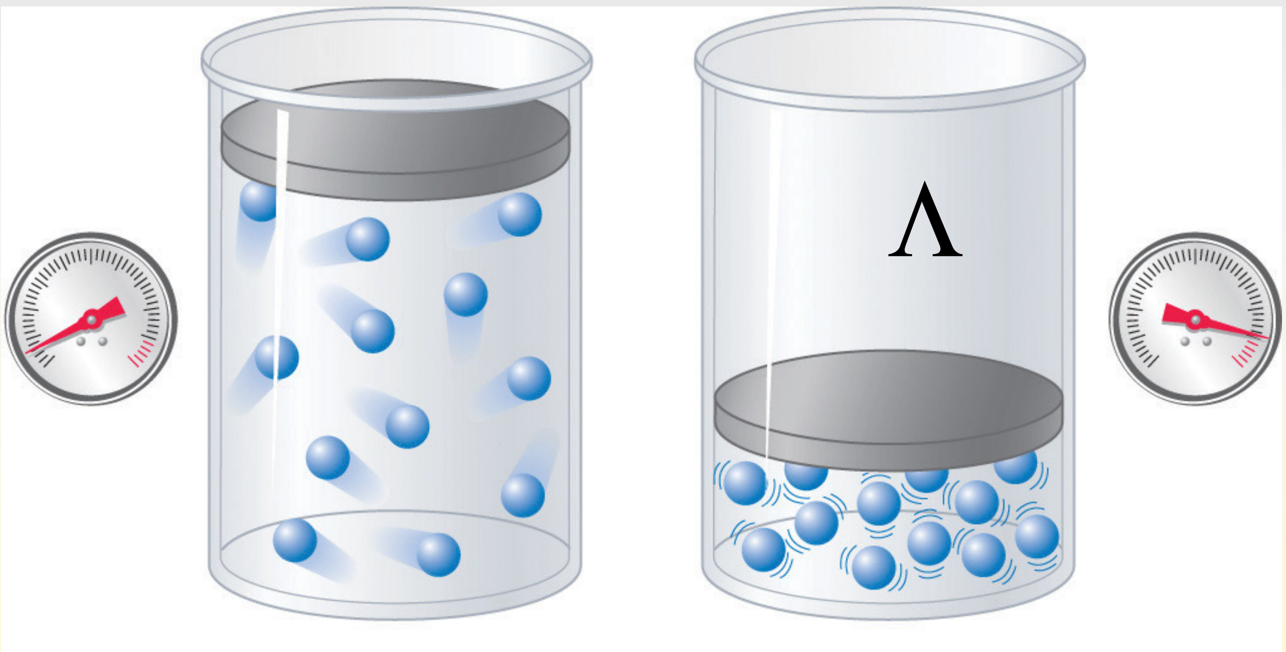
Stress-energy of matter

Dark Energy: $\Lambda > 0$ (cosmic tension)

String Theory: $\Lambda < 0$ (cosmic pressure)

Black Hole Chemistry:

regard vacuum energy as pressure



(a) Low pressure (b) High pressure

Smarr Formula

L. Smarr PRL 30, 71 (1973) [Err. 30, 521 (1973)].

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2$$

Schwarzschild Black hole $V = 1 - \frac{2M}{r}$

$$E = M = \frac{r_+}{2} \qquad T = \frac{1}{4\pi r_+} \qquad S = \pi r_+^2 \quad \Rightarrow \quad M = 2TS$$

Smarr ✓

Schwarzschild-AdS Black hole $V = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$ $\qquad \Lambda = -\frac{3}{l^2}$

$$E = M = \frac{l^2 + r_+^2}{2l^2} r_+ \qquad T = \frac{l^2 + 3r_+^2}{4\pi r_+ l^2} \quad \Rightarrow \quad M \neq 2TS$$

$$S = \pi r_+^2$$

~~Smarr~~ ?

Pressure from the Vacuum!

Thermodynamics | Gravity

Kastor/Ray/Traschen
CQG **26** (2009) 195001

Enthalpy $H \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \frac{\hbar\kappa}{2\pi}$ Surface gravity

Entropy $S \leftrightarrow \frac{A}{4\hbar}$ Horizon Area

Pressure $P \leftrightarrow -\frac{\Lambda}{8\pi G}$ Cosmological Constant

$$dH = TdS + VdP + \cdots \leftrightarrow dM = \frac{\kappa}{8\pi} dA + VdP + \cdots$$

First Law | First Law

$$H = E + PV + \cdots \leftrightarrow M = E - \rho V$$

Mass
= Total Energy
– Vacuum
Contribution
(infinite)

C. Teitelboim PLB **158** (1984) 293
J. Creighton and R.B. Mann, PRD **52** (1995) 4569
Caldarelli/Cognola/Klemm CQG **17** (2000) 399   T. Padmanabhan, CQG **19** (2002) 5387

Caldarelli/Cognola/ Klemm, CQG 17, 399 (2000)

Scaling Arguments

Suppose

$$f\left(\alpha^p x,\alpha^q y\right)=\alpha^r f\left(x,y\right) \Rightarrow rf\left(x,y\right)=p\frac{\partial f}{\partial x}x+q\frac{\partial f}{\partial y}y$$

In general $M\propto L^{D-3}$ $A\propto L^{D-2}$ $J_i\propto L^{D-2}$ $\Lambda\propto L^{-2}$ $Q\propto L^{D-3}$

$$M=M\left(A,J_i,Q,\Lambda\right)$$

$$\left(D-3\right)M=\left(D-2\right)\frac{\partial M}{\partial A}A+\left(D-2\right)\frac{\partial M}{\partial J_i}J_i+\left(D-3\right)\frac{\partial M}{\partial Q}Q-2\frac{\partial M}{\partial \Lambda}\Lambda$$

$$S=\frac{A}{4G} \qquad T=\frac{\kappa}{2\pi}=4G\frac{\partial M}{\partial A} \qquad \Phi_h=\frac{\partial M}{\partial Q} \qquad \left(\Omega_h-\Omega_\infty\right)_i=\frac{\partial M}{\partial J_i}$$

$$P=-\frac{\Lambda}{8\pi}=\frac{\left(D-2\right)\left(D-1\right)}{16\pi l^2} \qquad V=-8\pi\frac{\partial M}{\partial \Lambda}$$

$$\Rightarrow M=\frac{\left(D-2\right)}{\left(D-3\right)}TS-\frac{2}{\left(D-3\right)}VP+\frac{\left(D-2\right)}{\left(D-3\right)}\left(\Omega_h-\Omega_\infty\right)_i J_i+\Phi Q$$

First Law

$$\delta M = T_h \delta S_h + \sum_i (\Omega_h^i - \Omega_\infty^i)\delta J^i + V_h \delta P + \Phi \delta Q$$

Integrate

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega_h^i - \Omega_\infty^i) J^i - \frac{2}{D-2} P V_h + \frac{D-3}{D-2} \Phi Q$$

Example: D-dim'l Schwarzschild-AdS Black hole

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega_{D-2}^2$$

$$G_{ab} + g_{ab}\Lambda = 0 \Longrightarrow V = 1 - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{l^2}$$

$$\Lambda = \frac{(D-1)(D-2)}{2l^2}$$

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_{D-2}^2 \qquad V = 1 - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{l^2}$$

$$\omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$$

$$M = (D-2)\omega_{D-2}\frac{l^2 + r_+^2}{16\pi l^2} r_+^{D-3} \qquad S = \frac{\omega_{D-2}}{4} r_+^{D-2} \qquad J^i = 0$$

$$T = \frac{(D-3)l^2 + (D-1)r_+^2}{4\pi r_+ l^2} \qquad P = \frac{(D-2)(D-1)}{16\pi l^2} \qquad V = \frac{\omega_{D-2} r_+^{D-1}}{(D-1)}$$

$$\delta M = (D-2)\omega_{D-2}\left[\frac{(D-3)l^2 r_4^{D-4} + (D-1)r_+^{D-2}}{16\pi l^2}\delta r_+ - \frac{r_+^{D-1}}{8\pi l^3}\delta l\right]$$

$$T\delta S = \left[\frac{(D-3)l^2 + (D-1)r_+^2}{4\pi r_+ l^2}\right](D-2)\frac{\omega_{D-2}}{4} r_+^{D-3}\delta r_+$$

$$V\delta P = -\omega_{D-2} r_+^{D-1}\frac{(D-2)}{8\pi l^3}\delta l$$

$$\Rightarrow \quad \delta M = T\delta S + V\delta P$$

First Law ✓

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega_{D-2}^2 \qquad V = 1 - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{l^2} \qquad V(r_+) = 0$$

$$M = (D-2)\omega_{D-2}\frac{l^2 + r_+^2}{16\pi l^2} r_+^{D-3} \qquad S = \frac{\omega_{D-2}}{4} r_+^{D-2} \qquad J^i = 0 \qquad \omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$$

$$T = \frac{(D-3)l^2 + (D-1)r_+^2}{4\pi r_+ l^2} \qquad P = \frac{(D-2)(D-1)}{16\pi l^2} \qquad V = \frac{\omega_{D-2} r_+^{D-1}}{(D-1)}$$

$$\frac{D-3}{D-2} M = (D-3)\omega_{D-2}\frac{l^2 r_+^{D-3} + r_+^{D-1}}{16\pi l^2}$$

$$TS = \omega_{D-2}\frac{(D-3)l^2 r_+^{D-3} + (D-1) r_+^{D-1}}{16\pi r l^2}$$

$$-\frac{2}{D-2} P V_h = -\frac{2\omega_{D-2} r_+^{D-1}}{16\pi l^2}$$

$$\Rightarrow \quad \frac{D-3}{D-2} M = TS - \frac{2}{D-2} PV$$

Smarr ✓

Thermodynamic Volume

Thermodynamic $V \equiv \left(\frac{\partial M}{\partial P}\right)_{S,Q,J,\ldots}$

4D SAdS BH

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \qquad f = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$$

$$\Rightarrow \quad V = \frac{4\pi r_+^3}{3}$$

In general V is not the geometric volume of the black hole

4D Kerr-AdS BH

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Sigma = 1 - \frac{a^2}{l^2}\cos^2\theta$$

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - \frac{a\sin^2\theta d\varphi}{1-a^2/l^2}\right]^2 + \frac{\Sigma\sin^2\theta}{\rho^2}\left[adt - \frac{\left(r^2+a^2\right)d\varphi}{1-a^2/l^2}\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Sigma}d\theta^2$$

$$\Delta = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2mr$$

$$V = \frac{r_+ A}{3} + \frac{4\pi a J}{3}$$

Thermodynamic volume

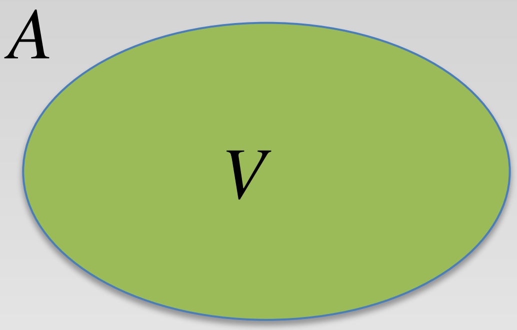
$$A = 4\pi l^2 \frac{a^2 + r_+^2}{l^2 - a^2}$$

Horizon area

Meaning of Thermodynamic Volume?

Q: What is the smallest area that encloses a given Euclidean volume V?

A: A spherical surface



$$R = \left(\frac{(D-1)V}{\omega_{D-2}}\right)^{1/D-1}\left(\frac{\omega_{D-2}}{A}\right)^{1/D-2} \leq 1$$

$$\omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$$

Black Holes: $R_{BH} \equiv \left(\frac{(D-1)V_{BH}}{\omega_{D-2}}\right)^{1/D-1}\left(\frac{\omega_{D-2}}{A_{BH}}\right)^{1/D-2}$

Conjecture: All black holes obey the *Reverse Isoperimetric Inequality* $R_{BH} \geq 1$

Cvetic/Gibbons/Kubiznak/Pope PRD**84** (2011) 024037

4D Kerr-AdS BH

$$R_{BH} \equiv \left(\frac{3V}{4\pi}\right)^{1/3}\left(\frac{4\pi}{A}\right)^{1/2} = \sqrt[3]{\frac{r_+}{l}\frac{r_+^2+a^2}{l^2-a^2}\left(1+\frac{a^2\left(r_+^2+l^2\right)}{2r_+^2(l^2-a^2)}\right)}\sqrt{\frac{l^2-a^2}{r_+^2+a^2}} \geq 1$$

$a = 0$

$\rightarrow 1$

Not always obeyed!

→ For a given thermodynamic volume, the entropy of a black hole is maximized by the Schw-AdS sol'n

Outline From Here

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