Holography of Astronomic Black Hole?!

Bin Chen (陈斌)

Peking University

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Black hole: harmonic oscillator in 21st Century?!

Black hole: simple system but rich physics

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$$S = \frac{A}{4}$$

Holographic nature of quantum gravity!
Q: how to understand the BH entropy microscopically?

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In this talk, I will focus on the holographic picture of black hole.

Lessons from string theory

One of the greatest achievements in string theory: for a class of 5D extremal charged BH, there exists microscopic counting Strominger and Vafa (1996).
It relies on string technology: D-branes configuration (D1-D5-P)

1. Supergravity BPS solution: has entropy
2. Open string point of view: moduli space of the D-brane configuration could be described by a 2D CFT
3. In other words, the D-brane configuration corresponds to a highly excited states in 2D CFT

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The study of physics of D3-branes led to Maldacena 1997
AdS/CFT: Quantum gravity in AdS_5 is dual to a CFT_4 at $\partial\mathrm{AdS}_5$

The near-horizon geometries of D-brane configurations include a AdS_3 factor, suggesting $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence.

Concrete realizations of holographic principle.

Can we “derive” holography without string theory?

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In the following, I will review the first approach and try to answer the question: do we have a holographic description of an astronomic black hole in the universe?

No SUSY, not related to string theory directly, ...

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$$\text{ASG} = \frac{\text{Allowed Symmetry Transformation}}{\text{Trivial Symmetry Transformation}}$$

"Allowed": the asymp. Killing symmetry $\hat{\chi}$ such that $\mathcal{L}_{\hat{\chi}} g_{\mu\nu} \sim h_{\mu\nu}$.
"Trivial": some of $\hat{\chi}$ lead to vanishing global charges.
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One gravity theory may have multiple sets of "good" boundary conditions, leading to different ASG.

$\text{AdS}_3/\text{CFT}_2$ correspondence

Recall: the near-horizon geometry of D1-D5-p configuration includes a AdS_3 factor. Let's get rid of string theory, and study AdS_3 gravity directly. A. Strominger (1997)

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In 1986, Brown and Heanneaux found that under appropriate boundary conditions the asymptotic symmetry group (ASG) of AdS_3 Einstein gravity is generated by two copies of Virasoro algebra with central charges

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For AdS_3 gravity, there exists another set of boundary conditions, the so-called CSS B.C., leading to the ASG generated by an Virasoro-Kac-Moody algebra. Compere et.al. (2013) $\Rightarrow$ $AdS_3/WCFT_2$

Remarks on ASG

1. Provides a novel way to understand AdS/CFT without string theory

2. Shows that boundary conditions could be essential in defining a quantum gravity theory.

3. It has led to a few remarkable developments in the past 15 years:
   - Warped AdS/CFT correspondence W. Li et.al (2008)
   - Kerr/CFT correspondence (holography in astronomy?) M. Guica et.al. (2008), ...
   - Flat space holography Bondi et.al. (1962), Sachs(1962), Barnich et.al. (2010),...
   - Celestial holography Pasterski et.al. (2017),...
   - ......

The holographic pictures are all related to 2D field theories with conformal(-like) symmetry.

2D CFT

- A conformal field theory is a quantum field theory with conformal invariance.
- In 2D, the conformal symmetry is infinitely dimensional, and therefore very powerful.
- Even though the detailed construction of the CFT dual to the black hole is not known, we can still read some universal properties from the symmetry.
- The most important ones for our later discussion are the Cardy formula and the 2-pt correlation function.
- One remarkable feature of 2D CFT is that the left-moving and right-moving sectors are completely decoupled, each having its own central charge and temperature.

Black hole in AdS/CFT

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This picture is most clear in $\mathrm{AdS}_3/\mathrm{CFT}_2$
1. A BTZ BH corresponds to the holographic CFT at finite temperature, whose degeneracy microscopically count the Bekenstein-Hawking entropy. The degeneracy could be read from Cardy's formula (in canonical ensemble)

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R)$$

where

$$T_L = \frac{r_+ + r_-}{2\pi}, \qquad T_R = \frac{r_+ - r_-}{2\pi}.$$

2. For BTZ black hole: Birmingham and Sachs (2002)

$$\omega_L = -k - 4\pi i T_L(n + h_L), \qquad \omega_L = k - 4\pi i T_R(n + h_R), \qquad n \in \mathbf{N}.$$

Moreover, one may compare the 2-pt retarded correlators directly.

The interesting thing is that there is a similar picture for 4D Kerr blackhole as well.

Kerr black holes

A Kerr black hole is characterized by the mass M and angular momentum $J = aM$. It could be described by the metric of the following form

$$ds^2 = -\frac{\Delta}{\hat{\rho}^2}(d\hat{t} - a\sin^2\theta d\hat{\phi})^2 + \frac{\sin^2\theta}{\hat{\rho}^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t}\right)^2 + \frac{\hat{\rho}^2}{\Delta}d\hat{r}^2 + \hat{\rho}^2 d\theta^2,$$

with

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2, \qquad \hat{\rho}^2 = \hat{r}^2 + a^2\cos^2\theta,$$

where we have used the unit $G = \hbar = c = 1$.

- Two horizons: $r_\pm = M \pm \sqrt{M^2 - a^2}$;
- The Hawking temperature, the angular velocity of the horizon and the entropy of the Kerr black hole are

$$T_H = \frac{r_+ - r_-}{8\pi M r_+}, \qquad \Omega_H = \frac{a}{2Mr_+}, \qquad S_{BH} = 2\pi M r_+.$$

Kerr/CFT correspondence

- Conjecture: A Kerr black hole could be holographically described by a 2D CFT with

$$c_L = c_R = 12J/\hbar, \qquad T_L = \frac{(r_+ + r_-)^2}{8\pi J}, \quad T_R = \frac{r_+^2 - r_-^2}{8\pi J};$$

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  2. Superradiant scattering and the low-frequency scattering with the hidden conformal symmetry: the amplitudes are in agreement with the CFT predictions.
- Feature: The set up of Kerr/CFT has nothing to do with string theory, and beyond AdS/CFT

How to set up the holographic description?

Central charges: read from Asymptotic Symmetry Group(ASG) analysis of NHEK geometry M.Guica et.al. 0809.4266

The first step: consider the near-horizon geometry of an extremal Kerr black hole (NHEK) J.M. Bardeen and G.T. Horowitz (1999)

$$ds^2 = 2J\Gamma\left(-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2(d\phi + rdt)^2\right),$$

where $\Gamma(\theta) = \frac{1+\cos^2\theta}{2}, \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$. For fixed θ, it is a warped AdS_3, as a $U(1)$ bundle on AdS_2, so has $SL(2,R)_R \times U(1)_L$ isometry group.

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Under a certain set of boundary condition, the $U(1)_L$ get enhanced into a Virasoro algebra with central charge $c_L = 12J$. M.Guica et.al. 0809.4266
For the extreme case, the dual temperature could be read from the Frolov-Thorne vacuum: $T_L = 1/2\pi$.

Perfect match of the macroscopic entropy of the black hole with the microscopic (CFT) entropy computed by the Cardy formula.
It looks like the dual CFT is a chiral one with vanishing right temperature.

Kerr/CFT for nonextreme BH

Actually, the dual CFT is not chiral. Its right-moving sectors describe the deviation from the extreme.

Nonextreme Kerr/CFT: the central charges are still $c_L = c_R = 12J$ even far away from extremality.

For the generic non-extreme case, the temperatures are read from the hidden conformal symmetry(HCS) in the low frequency scattering of the probe A.Castro et.al. 1004.0996

$$T_L = M^2/2\pi J, \qquad T_R = \sqrt{M^4 - J^2}/2\pi J.$$

Both the entropy counting and the low-frequency scattering amplitude support the conjecture

Scalar scattering

Let us consider the complex scalar field with mass μ scattering with the Kerr black hole. The Klein-Gordon equation is

$$\nabla_\mu \nabla^\mu \Phi - \mu^2 \Phi = 0.$$

Ansatz: $\Phi = e^{-i\omega t + im\phi} \mathcal{R}(r) \mathcal{S}(\theta)$.

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$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \mathcal{S} \right) + \left(\Lambda_{lm} - a^2(\omega^2 - \mu^2)\sin^2\theta - \frac{m^2}{\sin^2\theta} \right) \mathcal{S} = 0.$$

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The radial part of the wave function is of the form

$$\partial_r (\Delta \partial_r \mathcal{R}) + V_R \mathcal{R} = 0$$

with

$$\begin{aligned} V_R &= -\Lambda_{lm} + 2am\omega + \frac{H^2}{\Delta} - \mu^2(r^2 + a^2), \\ H &= \omega(r^2 + a^2) - am. \end{aligned}$$

Low-frequency limit and "Near" region

In the low frequency limit,

$$\omega M << 1,$$

the ω^2 term in the angular equation could be neglected.
To simplify our discussion, we focus on the massless scalar such that the angular equation is just the Laplacian on the 2-sphere with the separation constants taking values $\Lambda_{lm} = l(l+1)$.

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In the "Near" region,

$$r\omega << 1,$$

the radial equation could be simplified even more, and more importantly, it could be written in terms of $SL(2,R)$ quadratic Casimir.

Conformal coordinates for non-extremal BH

Let's introduce the conformal coordinates

$$\begin{aligned}
\omega^+ &= \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_R \phi + 2 n_R t}, \\
\omega^- &= \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_L \phi + 2 n_L t}, \\
y &= \sqrt{\frac{r_+-r_-}{r-r_-}}\, e^{\pi (T_L + T_R)\phi + (n_L + n_R) t},
\end{aligned}$$

Define locally the vector fields

$$\begin{aligned}
H_1 &= i\partial_+ \\
H_0 &= i\left(\omega^+\partial_+ + \frac{1}{2} y\partial_y\right) \\
H_{-1} &= i(\omega^{+2}\partial_+ + \omega^+ y\partial_y - y^2\partial_-)
\end{aligned}$$

which obey the $SL(2,R)$ Lie algebra: $[H_0, H_{\pm 1}] = \mp i H_{\pm 1}$.
Similarly we can define another set of vector fields $(\tilde{H}_0, \tilde{H}_{\pm 1})$ with $+ \leftrightarrow -$.

For extremal BHs, a new set of conf. coordinates BC et.al. 1007.4269

Casimir

The quadratic Casimir is

$$\begin{aligned}\mathcal{H}^2 = \tilde{\mathcal{H}}^2 &= -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) \\ &= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_- .\end{aligned}$$

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$$= \frac{1}{4}(y^2 \partial_y^2 - y\partial_y) + y^2 \partial_+ \partial_- .$$

The key point: the scalar radial equation is just the $SL(2,R)$ Casimir

$$\tilde{\mathcal{H}}^2 \mathcal{R}(r) = \mathcal{H}^2 \mathcal{R}(r) = l(l+1)\mathcal{R}(r),$$

with the following identifications:

$$n_R = 0, \qquad n_L = -\frac{1}{4M},$$
$$T_R = \frac{r_+ - r_-}{4\pi a}, \qquad T_L = \frac{r_+ + r_-}{4\pi a}.$$

Remarks

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- The $SL(2,R) \times SL(2,R)$ symmetry is spontaneously broken down to $U(1)_L \times U(1)_R$ subgroup by the periodic identification

$$\phi \sim \phi + 2\pi.$$

- Actually the conformal coordinates used here has been discussed in the case of BTZ black hole
- The relation between conformal and Boyer-Lindquist coordinates is reminiscent of the relation between Minkowski and Rindler coordinates in flat spacetime
- The periodic identification suggest that the Kerr black hole should be dual to a finite (T_L, T_R) mixed state in the dual CFT

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- This provides an effective way to determine (T_L, T_R) in CFT, and amazingly the Cardy formula gives the correct BH entropy
- Moreover the scattering amplitude is in perfect match with CFT predictions

Short summary

The holographic pictures of various black holes had been investigated intensely since the seminal work by M. Guica, T. Hartman, W. Song and A. Strominger in 2008.

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2. The superradiant scattering off near-extreme BH, or the low-frequency scattering off the generic BH $\Leftarrow$ (hidden) conformal symmetry

Remarkably, we have proposed a novel way to study the holographic pictures of the black holes by using the dynamics of both the outer and inner horizons. All the known results have been reproduced successfully.

BC et.al. 1206.2105,1208.4413,1212.1959,1212.1960,1301.0429

Where do the microstates reside in the bulk?

1. lie inside the event horizon of BH?
2. lie within a few ℓ_P's of the horizon? membrane paradigm W.H. Zurek and K.S. Thorne (1985), K.S. Thorne et.al. (1986)
3. occupies a region over a few R_s outside the horizon?

From the study of BH in string theory, greybody factors are essential to find the agreement between the microscopic and macroscopic scattering.

J. Maldacena and A. Strominger (1997)

The recent study on the information loss problem suggests that this region is important. G. Penington (2020), A. Almheiri et.al. (2019), A. Almheiri et.al.(2020)

Can we have a holographic description of a region outside the horizon, say "photon ring" ?

In the past few years, the study of black hole physics has entered a new era.

It was triggered by the direct detection of gravitational waves generated in binary BH merging, by LIGO in 2015.

Another important breakthrough is from BH image, taken successfully by Event Horizon Telescope (EHT) in 2017.

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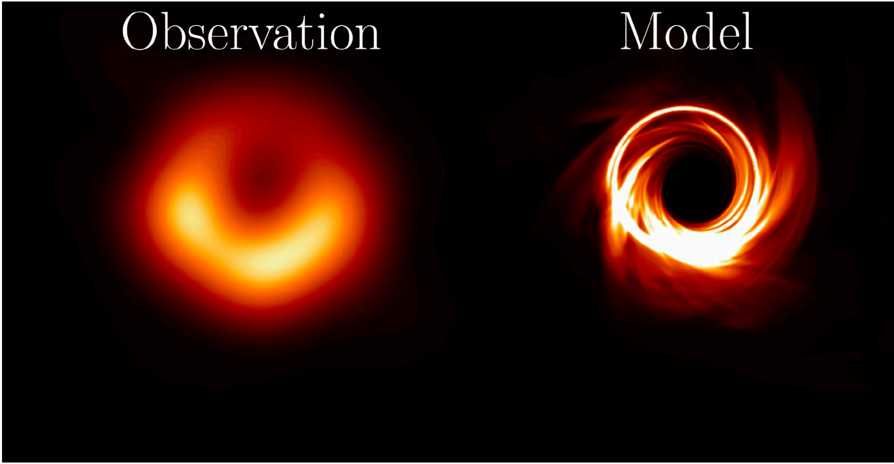
Provides direct evidence on the existence of black hole.

Test GR in strong-field regime and most dynamic area.

Features of BH image

1. Shadow: its boundary does not correspond to the event horizon of BH, but correspond to the trapped radius of photon
2. Photon ring
3. Asymmetry: Doppler effect



Photon shell

Photon shell: is the region of a black hole spacetime containing bound null geodesics or "bound orbits" that neither escape to infinity nor fall across the event horizon.
For Sch. BH, it is the 2D sphere at $r = 3M$.
For Kerr BH, it fattens to a 3D spherical shell:

$$r_-^\gamma \leq r \leq r_+^\gamma, \quad \theta_- \leq \theta \leq \theta_+, \quad 0 \leq \phi \leq 2\pi.$$

The inner circular equatorial orbit at r_-^γ is prograde, while the outer one at r_+^γ is retrograde.

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Photon shell: is the region of a black hole spacetime containing bound null geodesics or "bound orbits" that neither escape to infinity nor fall across the event horizon.
For Sch. BH, it is the 2D sphere at $r = 3M$.
For Kerr BH, it fattens to a 3D spherical shell:

$$r_-^\gamma \leq r \leq r_+^\gamma, \quad \theta_- \leq \theta \leq \theta_+, \quad 0 \leq \phi \leq 2\pi.$$

The inner circular equatorial orbit at r_-^γ is prograde, while the outer one at r_+^γ is retrograde.

The bound geodesics are unstable. The observed photon ring image arises from photons traveling on such "nearly bound" geodesics, which are referred to as the unstable fundamental photon orbits (UFPO).
Geodesic 1: bound orbit at r_0
Geodesic 2: initially $r = r_0 + \delta r_0$, after n half orbits

$$\delta r_n = e^{\gamma_L n} \delta r_0,$$

where γ is the so-called Lyapunov exponent.

Photon ring

The photon ring is the image on the observer screen produced by photons on nearly bound geodesics. In the limit in which the photons become fully bound, their images approach a closed curve C_γ.

The photon ring can be subdivided into subrings arising from photons that have completed n half-orbits between their source and the screen. the subrings occupy a sequence of exponentially nested intervals centered around C_γ.

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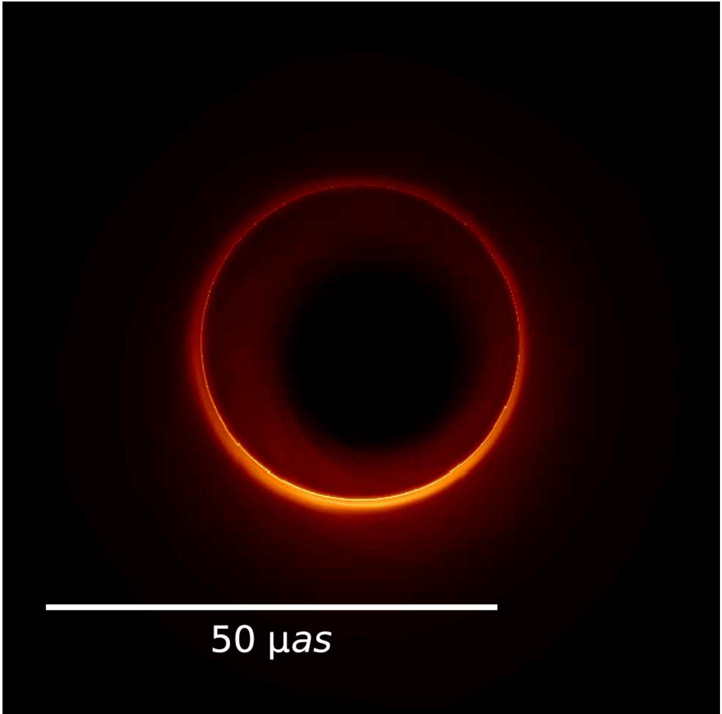
The subring substructure displays highly intricate but universal properties, which are insensitive to the detailed composition of the black hole atmosphere. Gralla et.al. (2019), M.D. Johnson et.al. (2020),...
This intricate structure is potentially measurable and is a prime target for future space-based VLBI missions. M.D. Johnson et.al. (2020), S.E. Gralla and A. Lupsasca (2020)....


50 μas

UFPO and high-frequency QNMs

The correspondence between high-frequency (eikonal approx.) of QNM and UFPO: the real part of the mode's frequency relates to the geodesic's orbital frequency, and the imaginary part of the frequency corresponds to the Lyapunov exponent of the orbit. Mashhoon (1985), V. Cardoso, et.al. 0812.1806, H. Yang, et.al. 1207.4253

$$\omega_{\ell mn} \simeq \left(\ell + \frac{1}{2}\right)\Omega(\mu) - i\gamma_L(\mu)(n + 1/2), \qquad \ell >> 1,$$

where γ_L is the Lyapunov exponent of the orbit, and $\mu = m/\ell$ is kept fixed as both ℓ and $|m|$ are taken large,

$$\Omega(\mu) = \frac{\mu}{\tilde{\lambda}(r_0(\mu))}$$

In a geometrical optics approximation, QNMs can be interpreted as particles trapped at unstable null geodesics and slowly leaking out.

Near-ring region

For each orbital radius $r_0 \in [r_-^\gamma, r_+^\gamma]$ of bound orbit, and radial deviation $\delta r = r - r_0$,

$$\text{Near-ring region:} \begin{cases} |\delta r| << M & \text{(near peak)}, \\ \left|\frac{m}{\omega_R} - \tilde{\lambda}\right| << M & \text{(near-critical in } m \text{ and } \phi), \\ \left|\frac{A_{\ell m}^R - m^2}{\omega_R^2} - \tilde{\eta}\right| << M & \text{(near-critical in } \ell \text{ and } \theta), \\ \frac{1}{\omega_R} << M & \text{(high frequency)}. \end{cases}$$

where $A_{\ell m}^R$ is the real separation constant, m is the angular momentum, ω_R is the energy of null geodesics, $\tilde{\lambda}, \tilde{\eta}$ are energy-rescaled angular momentum and Carter constant fixed by the orbital radius.

Conformal symmetry in QNM

Under the eikonal limit and in the near-ring region, the radial equation of QNM becomes an inverted harmonic oscillator eigenvalue problem,

$$\mathcal{H}\psi = i\omega_I \psi,$$

which suggests that there is a $SL(2,R)_{QN}$ symmetry,

$$\{a_\pm\}, \qquad L_0 = \frac{i}{4}(a_+ a_- + a_- a_+) = \frac{i}{2\gamma_L}\mathcal{H}, \qquad L_\pm = \pm\frac{a_\pm^2}{2},$$

with $[a_+, a_-] = i$. S. Hadar, et.al. 2205.05064

QNM as h.w.r. of $SL(2, R)_{QN}$

The quasinormal modes correspond to the highest-weight representations of $sl(2, R)_{QN}$: D. Birmingham, et.al. (2002), BC et.al. (2009)

$$\begin{aligned} L_0\psi_h &= h\psi_h, \quad \omega_I = -2\gamma_L h, \\ L_+\psi_h &= 0, \quad \text{(Ingoing B.C.)} \end{aligned}$$

However, there are two sets of fundamental modes with $h = 1/4$ and $h = 3/4$ respectively S. Hadar, et.al. 2205.05064

$$\begin{aligned} \Phi_{1/4} &= e^{-\frac{1}{2}\gamma_L t}\psi_{1/4}(x), \qquad \psi_{1/4}(x) = e^{\frac{i}{2}i\gamma\omega_R x^2}, \\ \Phi_{3/4} &= e^{-\frac{3}{2}\gamma_L t}\psi_{3/4}(x), \qquad \psi_{3/4}(x) = xe^{\frac{i}{2}i\gamma\omega_R x^2}. \end{aligned}$$

Higher overtones are obtained as $sl(2, R)_{QN}$-descendants:

$$\Phi_{h,N} = L_-^N \Phi_h(t, x).$$

It seems that the QNM overtones in the Kerr near-ring region fall into two irr. reps. of $sl(2, R)_{QN}$.

Novel realization of $SL(2,R)_{QN}$

In 2212.02958, we proposed a novel way to realize $SL(2,R)_{QN}$, and solve the above 'two-set' problem. Roughly, we noticed that

$$L_+ = sa_+, \qquad L_0 = a_-a_+ + h, \qquad L_- = s^{-1}(a_-^2 a_+ + 2ha_-),$$

with s being a nonzero constant, generate an $sl(2,R)$ algebra. With this realization, we find that the QNM overtones in the near-ring region fall into one irr. h.w. rep. of $sl(2,R)_{QN}$

Phase space of photon ring

Consider the 6-dim. phase space of null geodesics in Kerr, spanned by $(r, \theta, \phi, p_r, p_\theta, p_\phi)$ with canonical symplectic form.

Conserved quantities:
Hamiltonian $H = -p_t$, Carter constant Q, angular momentum $L = p_\phi$.

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With a coordinate transformation $(r, \theta, \phi, p_r, p_\theta, p_\phi) \to (T, \Phi, \Psi, H, L, Q + L^2)$, which preserves the symplectic form, one may find the following trivial equations of motion

$$\dot{H} = 0, \quad \dot{L} = 0, \quad \dot{Q} = 0,$$
$$\dot{\Psi} = 0, \quad \dot{\Phi} = 0, \quad \dot{T} = 1,$$

suggesting T conjugate to energy.

The bound photon orbit at $r_0(L, Q)$ has a conserved energy $H_0(L, Q)$. Define $\hat{H} = H - H_0$, and consider the unbound geodesics with $\hat{H} < 0$ that begins and ends at null infinity.

Emergent conformal symmetry of the photon ring

Since $(T, \Phi, \Psi, H, L, Q + L^2)$ are canonical, the functions

$$H_+ = \hat{H}, \qquad H_0 = -\hat{H}T, \qquad H_- = \hat{H}T^2,$$

obey the $sl(2, R)_{PR}$ algebra.
This algebra commutes with L and Q and therefore acts within superselection sector of fixed L and Q. But the flow generated by H_0 change the energy $H = \hat{H} + H_0$, and therefore acts on the impact parameters as well as the radius of the orbit.

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The $SL(2, R)_{PR}$-invariant locus in phase space is the photon shell. The large finite dilation

$$\hat{H}(\alpha) = e^{-\alpha}\hat{H}(0)$$

scale down $\hat{H}$ and scale up T, pushing any trajectory asymptotically onto the homoclinic orbits at large times.

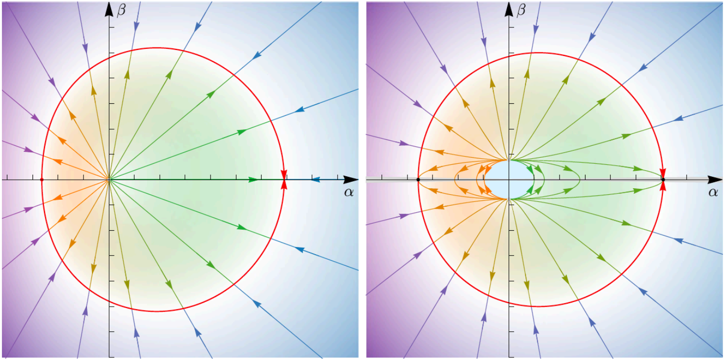

Figure: Action of dilations on the image plane of an observer at a large distance, located in the equatorial plane (first panel) or at an inclination of 60 degrees (second panel). From S. Hadar et.al. 2205.05064

The dilation

$$D_0 = e^{-\gamma_L H_0},$$

takes $n_{orb} \to n_{orb} + 1$.

Conclusion

The holographic pictures of astrophysical BH have been studied for many years.
Kerr/CFT: a Kerr BH could be holographically described by a 2D CFT.

This picture was recently generalized to holography of the photon rings.
In the near-ring regions, there are emergent conformal symmetries
1. in high-frequency QNMs
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The emergent conformal symmetry is encouraging, but is not a decisive evidence for a holographic picture. There's a lot to do!