Emergent Gravity from the Entanglement Structure in Group Field Theory

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Introduction

- Quantum gravity: why and how Einstein theory of gravity should be quantized?
- Approach: non-perturbative quantization approaches. Group field theory (GFT) [Krajewski, 2011]
- Emergent gravity:
  - quantum entanglement [Banks, 2020; Dieks et al., 2015; Faulkner et al., 2013; Fukuma et al., 2003; Hubeny et al., 2007; Kitaev and Suh, 2019; Maldacena and Stanford, 2016; Maldacena and Susskind, 2013; Maldacena et al., 2016; Mertens et al., 2017; Saad et al., 2018; Van Raamsdonk, 2010];
  - quantum information [Banks, 2020; Dieks et al., 2015; Faulkner et al., 2022; Qi, 2018]

A emergent gravity model

Break down of the model introduced by Sung-Sik Lee in [Lee, 2020].

- Fields: Φ_x^A, A labels species called colors, x labels sites, $\langle \Phi_x^A | \Phi_y^B \rangle = \delta^{A,B}\delta_{x,y}$
- Conjugate momentum defined by Π_A^x, $[\Phi_x^A, \Pi_B^y] = i\delta_B^A \delta_x^y$.
- Hilbert space $\mathbb{H} = \bigotimes_x \mathbb{H}_x$, called a frame.
- Spacial Hamiltonian constraint $\hat{\mathcal{G}}_Y = \mathrm{tr}\{\hat{\mathcal{G}} Y\}$ with $\hat{\mathcal{G}}_y^x = \frac{1}{2}\left(\hat{\Pi}_A^x \hat{\Phi}_y^A + \hat{\Phi}_y^A \hat{\Pi}_A^x\right)$.
- Temporal Hamiltonian constraint $\hat{\mathcal{H}}_V = \mathrm{tr}\left[\left(-\hat{\Pi}\hat{\Pi}^T + \frac{\tilde{\alpha}}{M^2}\hat{\Pi}\hat{\Pi}^T\hat{\Phi}^T\hat{\Phi}\hat{\Pi}\hat{\Pi}^T\right)V\right]$
- A path-integral formula $\langle 0 | \chi \rangle = \int \mathcal{D}\Phi \mathcal{D}k \mathcal{D}v \mathcal{D}y e^{iS} \chi(\Phi, k)$,
  $S = -\int d\tau \mathrm{tr}[(-\phi(\tau)\partial k(\tau) - V(\tau)\mathcal{H}(\tau) - Y(\tau)\mathcal{G}(\tau)]$
- Collective variables, provided by k, decides the dimension, topology and geometry of the spacetime, hence the full spapcetime is emergent.

Group field theory

Functional $\Phi(\mathbf{g})$, $\mathbf{g} = \{g_1, g_2, \cdots, g_D\}$, D dimension of the spacetime. Action

$$S[\Phi] = K[\Phi] - V[\Phi]\,, \tag{1}$$

where $K[\Phi]$ is the kinetic term,

$$K[\Phi] = \frac{1}{2}\Big(\prod_i^D \int dg_i d\tilde{g}_i\Big)\Phi(g_i)\mathcal{K}(g_i\tilde{g}_i^{-1})\Phi(\tilde{g}_i)\,. \tag{2}$$

example for potential term

$$\frac{\lambda}{(D+1)!}\Big(\prod_{i\neq j=1}^{D+1}\int dg_{ij}\Big)\Phi(g_{1j})\cdots\Phi(g_{(D+1)j})\mathcal{V}(g_{ij}g_{ji}^{-1})\,. \tag{3}$$

$\Phi(\mathbf{g}, \chi)$, χ relational clock [Wilson-Ewing, 2019]. Peter-Weyl expansion

$$\Phi(\vec{g}) = \sum_{j_i, m_i, n_i, \iota} \phi^{\vec{j},\iota}_{\vec{m}} \mathcal{I}^{\vec{j},\iota}_{\vec{n}} \prod_{a=1}^{d+1} \frac{1}{d(j_a)} D^{j_a}_{m_a n_a}(g_a)\,, \tag{4}$$

action:

$$S[\phi] = \frac{1}{2} \sum_J \int d\chi \sum_{n=0}^{\infty} \phi_J(\chi) \mathcal{K}^{(2n)}_J \partial^{(2n)}_\chi \phi_J(\chi)\,, \tag{5}$$

$$\mathcal{K}^{(2n)}_J = \int d\delta\chi \frac{1}{(2n)!} \mathcal{K}_J((\delta\chi)^2)((\delta\chi)^2)^n\,. \tag{6}$$

$$\pi_J(\chi) = \frac{\delta S[\phi]}{\delta(\partial_\chi \phi_J(\chi))} = -\mathcal{K}^{(2)}_J \frac{\partial \phi_J(\chi)}{\partial \chi}\,, \tag{7}$$

Hamiltonian

$$\mathcal{H} = -\sum_J \left[\frac{\pi_J(\chi)^2}{2K^{(2)}_J} + K^{(0)}_J \frac{\phi_J(\chi)^2}{2} \right]. \tag{8}$$

GFT entanglement graph state (EG-state) [Colafranceschi and Oriti, 2021] Fock space:

$$\mathcal{F} = \bigoplus_{V=1}^{\infty} \mathrm{sym}(\mathcal{H}^1 \bigotimes \cdots \bigotimes \mathcal{H}^V) \,, \tag{9}$$

Projection operator:

$$\mathbb{P}_i^{x \bigotimes y} := \int dh_i^{xy} dg_i^x dg_i^y |g_i^x\rangle\langle g_i^x| \bigotimes |g_i^y\rangle\langle g_i^y h_i^{xy}| \,, \tag{10}$$

$$\mathbb{P}_i^{x \bigotimes y}|\psi\rangle = \int \prod_x d\mathbf{g}^x \int \Big(\prod_{l\in\gamma} dh_l\Big) \psi(\cdots, g_i^x h_l, \cdots, g_i^y h_l, \cdots) \bigotimes_x |\mathbf{g}^x\rangle \,. \tag{11}$$

EG-state

$$|\psi_\gamma\rangle = \prod_{x<y} \prod_{A_{xy}^i=1} \mathbb{P}_i^{x \bigotimes y}|\psi\rangle = \int \prod_x d\mathbf{g}^x \int \prod_{x<t_i(x)} d\mathbf{h}^{x\mathbf{t}(x)} \psi(g_i^x h_i^{xt_i(x)}) \bigotimes_x |\mathbf{g}^x\rangle \,, \tag{12}$$

The model

$\Phi(\mathbf{g}_x, \lambda_x)$, $\lambda_x = \sum_y \chi^i_x \chi^i_y$, enabling the correlations between vertices after introducing projection operators, Action:

$$S = \sum_J \int d\lambda_x d\delta\lambda_x \varphi_J(\lambda_x) K(\delta\lambda_x) \varphi_J(\lambda_x + \delta\lambda_x)\,, \tag{13}$$

where φ_J is a functional mode within the action, and $J = \{\mathbf{j}, \mathbf{m}, \iota\}$ is the collection of the representations j, the magnetic-numbers m on the semi-links, as well as the intertwiners ι on the vertices. Tylor expansion:

$$S = \int d\lambda_x \big(K^{(0)} \varphi_x^2 + K^{(1)} \varphi_x \partial_\lambda \varphi_x - \frac{1}{2} K^{(2)} \partial_\lambda \varphi_x \partial_\lambda \varphi_x \big)\,, \tag{14}$$

with

$$K^{(n)} = \int d(\delta\lambda) K(\delta\lambda) (\delta\lambda)^n\,. \tag{15}$$

The conjugate momentum of φ_x is

$$\pi_x \equiv \frac{\delta \mathscr{L}}{\delta(\partial_\lambda \varphi)} = K^{(1)}\varphi_x - K^{(2)}\partial_\lambda \varphi_x \,. \tag{16}$$

Legendre transform gives Hamiltonian:

$$H = \left(-K^{(0)} - \left(\frac{1}{2}+\alpha\right)\frac{(K^{(1)})^2}{K^{(2)}}\right)\varphi_x^2 + -\left(\frac{1}{2}+\alpha\right)\frac{\pi_x^2}{K^{(2)}} + (1+2\alpha)\frac{K^{(1)}}{K^{(2)}}\varphi\pi + \alpha K^{(2)}\partial_\lambda\varphi_x\partial_\lambda\varphi_x \,. \tag{17}$$

Momentum space:

$$H(k_x) = -\frac{1}{4K^{(2)}}\pi_x(k_x)^2 + \frac{K^{(2)}}{4}k_x\varphi(k_x)k_x\varphi(k_x) + \frac{K^{(1)}}{2K^{(2)}}\varphi(k_x)\pi(k_x) + \frac{K^{(2)}}{4}\left(-\frac{4K^{(0)}}{K^{(2)}} - \frac{(K^{(1)})^2}{(K^{(2)})^2}\right)\varphi_x^2 \,. \tag{18}$$

Emergent gravity

$\varphi_x = \sum_y \phi(\chi_x^i)\phi(\chi_y^i)$, $\phi_x^i = \phi(\chi_x^i)$, the conjugated momenta

$$\omega_x^i = \pi \sum_{y \neq x} \phi_y^i \,. \tag{19}$$

define $k_x = l \sum_y \omega_x^i \omega_y^i$, $1/\sum_y \phi_{y\neq x}^k \sum_{z\neq x} \phi_z^k = N_x$, set $l^2 = N_x$, then defining $N_x/K^{(2)}$ as the lapse function, the temporal Hamiltonian constraint then becomes

$$\frac{N_x}{4K^{(2)}} \left(-\omega_x^i \omega_x^i + (K^{(2)})^2 \sum_{y,z} \omega_x^i \omega_y^i \phi_y^j \phi_z^j \omega_z^k \omega_x^k \right) . \tag{20}$$

the third term (spacial Hamiltonian constraint)

$$\frac{K^{(1)}}{2K^{(2)}} \varphi(k_x)\pi(k_x) = \frac{K^{(1)}}{2K^{(2)}} \phi_x^i \omega_x^i \,, \tag{21}$$

This is the Hamiltonian in the emergent gravity model [Lee, 2020]

Path-integral and RG flow

the Schrödinger equation:

$$\hat{H}|\Psi_\Gamma\rangle = i\hbar \frac{d|\Psi_\Gamma\rangle}{d\lambda} \,. \tag{22}$$

For infinitesimal time-steps, the Schrödinger equation applied to the EG-states entails

$$|\Psi_\Gamma(\lambda + \delta\lambda)\rangle = e^{-i\hat{H}\delta\lambda/\hbar}|\Psi_\Gamma(\lambda)\rangle \,. \tag{23}$$

For an infinitesimal step

$$\langle\Psi_{\Gamma'}|\Psi_\Gamma\rangle = \langle\Psi_{\Gamma'};\lambda'|e^{-i\hat{H}\delta\lambda/\hbar}|\Psi_\Gamma;\lambda\rangle \,. \tag{24}$$

EG-state can be expressed by [Colafranceschi and Oriti, 2021]

$$|\psi_\Gamma\rangle = \sum_J \psi_J(\lambda_{xy}) \prod_{A_{xy}=1} \delta_{J_x J_y} |\theta_{\Gamma,J}(\{g_{xy}\});\lambda_{xy}\rangle \,, \tag{25}$$

$|\theta_{\Gamma,J}(\{g_{xy}\})\rangle$ basis elements of the graph states, A_{xy} adjacency matrix.

In momentum space

$$|\Psi_\Gamma\rangle = \int \mathcal{D}k e^{-ik^{xy}\lambda_{xy}} \psi(k_{xy}) \prod_{A_{xy}=1} |\theta_\Gamma(\{g_{xy}\}); k_{xy}\rangle \,. \tag{26}$$

Inserting virtual slices

$$\langle\Psi_{\Gamma'}|\Psi_\Gamma\rangle = \int \mathcal{D}k\mathcal{D}\theta \langle\Psi'_\Gamma|e^{-i\hat{H}\delta\lambda}|\theta^{(N-1)}; k^{(N-1))}\rangle \times \cdots \times \langle\theta^{(1)}; k^{(1)}|e^{-i\hat{H}\delta\lambda}|\Psi_\Gamma\rangle \,, \tag{27}$$

We also insert virtual slices labeled by $\lambda^{(n)}$ to evaluate tht path integral. The inner product among $|\theta_{\Gamma,J}(\{g_{xy}\})\rangle$'s equals 1 or vanishes; the inner product among $|\lambda^{(n)}\rangle$ showes time evolution. The path should be constrained by shift tensor N_s^{xy} and lapse tensor N_l^{xy}.

$$\mathrm{tr}\{N_l C_0\} = N_l^{xy}\left(-\frac{1}{4K^{(2)}}\pi_x\pi_y + \frac{K^{(2)}}{4}k_x\phi_x k_y\phi_y\right),$$
$$\mathrm{tr}\{N_s C\} = N_s^{xy}\frac{K^{(1)}}{2K^{(2)}}\pi_x\phi_y\,, \tag{28}$$

the path integral becomes

$$\langle\Psi_{\Gamma'}|\Psi_\Gamma\rangle = \int \mathcal{D}\lambda\mathcal{D}k\mathcal{D}N_s\mathcal{D}N_l\psi_\Gamma(k_{xy})\psi_{\Gamma'}(k'_{xy})e^{iS[N_s,N_l]}\,, \tag{29}$$

with the action

$$S[N_s, N_l] = \int -d\lambda\left(\sum_{xy} k_{xy}\partial_\lambda\lambda_{xy} - \mathrm{tr}\{CN_s\} - \mathrm{tr}\{C_0N_l\}\right) \tag{30}$$

For an infinitesimal step from λ_1 to λ_2, the action (30) can be recast as $S = S_2 - S_1$. Consider

$$|S_1\rangle = \int \mathcal{D}\lambda e^{S_1} |\lambda\rangle \,, \tag{31}$$

and act similarly for the final state $|S_2\rangle$. The Fourier transform of the state gives

$$|S_1(k)\rangle = \int \mathcal{D}k \mathcal{D}\lambda e^{S_1(k_{xy})} |k\rangle \,. \tag{32}$$

Their inner product is shown in [Lee, 2016] as a coarse grain in a Wilsonian RG flow, hence the path-integral provides an RG flow.

$\phi^i_x = \phi^i_{x,a} dx^a$, and make a correspondence $\phi^i_a \sim e^i_a$, $\varphi_x \propto g_{ij} e^i_a e^i_b = g_{ab}$ is the metric of the physical spacetime, and the Shrödinger equation then describes the flow of the metric, a Ricci flow.

Ashtekar formalism of GR

Foliating the spacetime in a series of time-slices, the Einstein-Hilbert action can be recast as [Arnowitt et al., 1959]

$$S = \int dt dx^3 \left(\pi^{ab}\dot{q}_{ab} - NC(\pi, q) - 2N^a C_a(\pi, q)\right) , \tag{33}$$

ADM form of the metric

$$ds^2 = -(N^2 - N_a N^a)dt^2 + 2N_a dx^a dt + q_{ab} dx^a dx^b \, . \tag{34}$$

$$C = G_{abcd}\pi^{ab}\pi^{cd} - \sqrt{q}R[q] \, , \tag{35}$$

$$C_a = D_a \pi^{ab} \, , \tag{36}$$

Using Ashtekar variables [Ashtekar, 1986]

$$A_a^i = \Gamma_a^i[e] + \beta k_a^i\,, \tag{37}$$

The conjugate momentum of A_a^i is the densitized triad,

$$E_a^i = |\det(e)| e_a^i\,. \tag{38}$$

the Hamiltonian constraints can be written as

$$H[N] = \int dx^3 N \epsilon_{ijk} F_{ab}^i E^{aj} E^{bk}\,, \tag{39}$$

$$D[\vec{N}] = \int dx^3 N^a F_{ab}^i E^{bi}\,, \tag{40}$$

where $F_{ab}^i = \partial_a A_b^i - \partial_b A_a^i + \epsilon^i_{jk} A_a^j A_b^k$ is the curvature of the Ashtekar connection.
Gauss constraint

$$G[\Lambda] = \int dx^3 \Lambda^i D_a E_i^a\,. \tag{41}$$

We propose the correspondence

$$\phi_a^i \sim E_a^i \,, \qquad \omega_a^i \sim A_a^i \,, \tag{42}$$

considering the background topology as a G-bundle, $\mathrm{SU}(2)$, we have

$$\begin{aligned}
\pi_x^2 &= \omega_x^i \phi_{x,i} \omega_x^j \phi_{x,j} / \varphi_x^2 \\
&= [\varpi_x^i \wedge \omega_x^j] \cdot [\phi_{x,i} \wedge \phi_{x,j}] / \varphi_x^2 \\
&= \epsilon^{ij}{}_k T^k \omega_{x,a}^i \omega_{x,b}^j \epsilon_{ijk} T^k \phi_{x,i}^a \phi_{x,j}^b / \varphi_x^2 \,,
\end{aligned} \tag{43}$$

$\omega_a \sim \partial_a \sim D_a$, $\epsilon^{ijk} \omega_a^i \omega_b^j = \epsilon^{ijk} \omega^i D_a \omega_b^j = F_{ab}^k$. The time constraint then becomes

$$H[N] = \int dr^3 \left(- \frac{1}{2K^{(2)} \varphi^2(\mathbf{r})} \right) \epsilon_{ijk} F_{ab}^k(\mathbf{r}) \phi^{ia}(\mathbf{r}) \phi^{ib}(\mathbf{r}) \,. \tag{44}$$

Expanding π_y around π_x, the spacial constraint

$$\begin{aligned} C_s &= \mathrm{tr}\{N^{xy}\phi_x^i(\omega_{x,i} + \partial_a\omega_{y,i}|_{y=x}(r_y^a - r_x^a))\} \\ &= \sum_{x,y}(N^{xy}\phi_x^i\omega_{x,i} + N^{xy}\phi_x^i\partial_a\omega_{y,i}|_{y=x}\delta r_{xy}^a) \\ &= \sum_x N_x\phi_x^i\omega_{x,i} + N_x^a\phi_x^i D_a\omega_{x,i}\,. \end{aligned} \tag{45}$$

$N_x = \sum_y N_{x,y}$ and $N_x^a = \sum_y N_{xy}\delta r_{xy}^a$,$\delta r_{xy}^a = (r_y^i - r_x^i)$, it is a sum over

$$D[\vec{N}] = \int dr^3 N^a(\mathbf{r})\phi_i^b(\mathbf{r})F_{ab}^i(\mathbf{r})\,, \tag{46}$$

which is the spatial Hamiltonian constraint in Eq. (40) and

$$\begin{aligned} G[\Lambda] &= \int dr^3 N(\mathbf{r})\omega_i(\mathbf{r})D_a\phi^{a,i}(\mathbf{r}) \\ &= \int dr^3 \Lambda_i(\mathbf{r})D_a\phi^{a,i}(\mathbf{r})\,, \end{aligned} \tag{47}$$

as the Gauss constraint.

Our work: Imaginary Hamiltonian

$$\hat{a}_J(k) = A_J(k)\varphi_J(k) + \frac{i}{2A_J}\hat{\pi}_J(k)\,, \tag{48}$$

$$\hat{a}^\dagger_J(k) = A_J(k)\varphi_J(k) - \frac{i}{2A_J}\hat{\pi}_J(k)\,, \tag{49}$$

Hamitlonian (18) becomes

$$\hat{H} = \sum_J \left(\mathrm{sgn}(K_J^{(2)}) \frac{|\varpi_J(k)|}{2} (\hat{a}^\dagger_J \hat{a}_J + \frac{1}{2}) + \frac{i}{2}((\hat{a}^\dagger_J)^2 - \hat{a}_J^2 + 1) \right)\,. \tag{50}$$

Complex action theory (CAT) [Nagao and Bech Nielsen, 2015; Nagao and Nielsen, 2011a,b, 2012, 2013, 2017a,b, 2022; Nielsen and Ninomiya, 2006].

Periodic CAT introduced in [Nagao and Nielsen, 2022], related to our work: as the entanglement, i.e., the relational clock undergoes a monotonously increase, and the EG-states evolve periodically, implying a cyclic universe.

Summary

- Emerges from the entanglement structure of Group Field Theory, following a RG flow that is generated by the Schrödinger equation of a system with non Hermitian Hamiltonian;
- Entails the Ashtekar formulation of gravity at the scale defined by the mass term in the Hamiltonian;
- Provides a straightforward link among general relativity and quantum mechanics;
- Evade the problem of time and the problem of nonrenormalzability of quantum gravity.

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