The scrambling power of gravity for black hole radiation

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Background

- Black hole radiation (Hawking radiation): black hole radiates like a black body, and this thermal radiation can be described in the framework of the quantum field theory in curved space-time.

- Information loss paradox: a pure state becomes a mixed state after the black hole evaporation, this implies a loss of unitary.

- It is now generally believed that the information is preserved, and the spectrum just looks thermal.

- Including soft hair: including the correlation between hard and soft particles (their energies are too small to be seen by detector)

Quantum field theory in collapsing space-time

- i^+ : the future time-like infinity
- I^+ : the future null infinity
- H^+ : the (future) black hole horizon
- i^- : the past time-like infinity
- I^- : the past null infinity

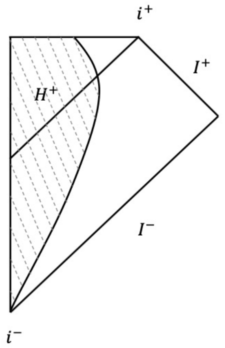

Figure: The Penrose diagram of black hole formation

Quantum electrodynamics in collapsing space-time

The expansions of ψ^{in} or ψ^{out} in terms of the two different modes are

$$\begin{aligned}\psi^{in/out} &= \sum_Q \left\{\psi_Q^{1+}(a_Q^1)_{in/out} + \psi_Q^{1-}(b_Q^{1\dagger})_{in/out}\right\} \\ &= \sum_Q \left\{\psi_Q^{2+}(a_Q^2)_{in/out} + \psi_Q^{2-}(b_Q^{2\dagger})_{in/out} + \psi_Q^{3+}(a_Q^3)_{in/out} + \psi_Q^{3-}(b_Q^{3\dagger})_{in/out}\right\}. \qquad (1)\end{aligned}$$

These two different mode expansions define four inequivalent vacuum states

$$(a_Q^1)_{in}|0\rangle_{in} = (b_Q^1)_{in}|0\rangle_{in} = 0, \ \forall Q, \qquad (2)$$

$$(a_Q^2)_{in}|\bar{0}\rangle_{in} = (a_Q^3)_{in}|\bar{0}\rangle_{in} = (b_Q^2)_{in}|\bar{0}\rangle_{in} = (b_Q^3)_{in}|\bar{0}\rangle_{in} = 0, \ \forall Q, \qquad (3)$$

$$(a_Q^1)_{out}|0\rangle_{out} = (b_Q^1)_{out}|0\rangle_{out} = 0, \ \forall Q, \qquad (4)$$

$$(a_Q^2)_{out}|\bar{0}\rangle_{out} = (a_Q^3)_{out}|\bar{0}\rangle_{out} = (b_Q^2)_{out}|\bar{0}\rangle_{out} = (b_Q^3)_{out}|\bar{0}\rangle_{out} = 0, \ \forall Q. \qquad (5)$$

Mutual information on hard and soft degrees of freedom

- $A,\ B \xrightarrow{U} C,\ D$

- $A,\ B \xrightarrow{U} C,\ D,\ E,\ F$

- Combine the three kinds of d.o.f. (D, E, and F) as one kind of d.o.f. (D).

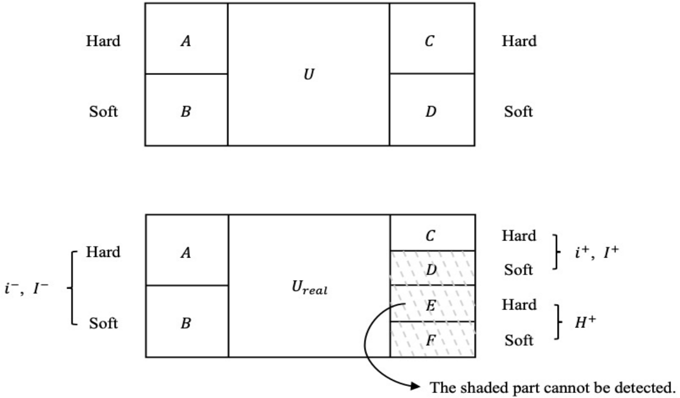

Figure: Scattering with soft photon in different spacetime

Mutual information on hard and soft degrees of freedom

- $U_{AB\to CD} \longrightarrow |U_{AB\to CD}\rangle \longrightarrow \rho_{ABCD}$
- $I_3(A:C:D)$
  $= I(A:C) + I(A:D) - I(A:CD)$
  $= S(\rho_C) + S(\rho_D) - S(\rho_{AC}) - S(\rho_{AD})$
- $S(\rho) = -\operatorname{tr}(\rho \log \rho) \longrightarrow I_3$
  $S_2(\rho) = -\log \operatorname{tr} \rho^2 \longrightarrow I_{3(2)}$
- $-2 \log d_A \leq I_3 \leq I_{3(2)} \leq 0$

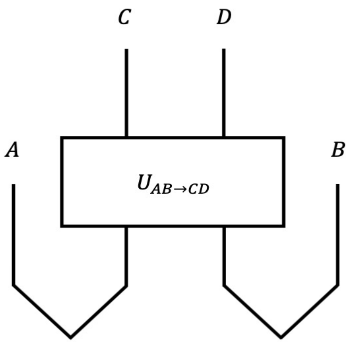

Figure: Doubled state of a unitary operator U

Tripartite mutual information in collapsing space-time

The amplitudes in collapsing space-time can be divided into two factors, one is Bogolubov coefficient, and the other is the amplitude formed by two Fock spaces using the same modes.

Thus, the relevant unitary operator U_{real} can be rewritten as

$$U_{real} = U_0 U = (\mathbb{I} - iG_0)(\mathbb{I} - iT) = \mathbb{I} - iG_0 - iT. \tag{6}$$

The definitions of U and U_0 are

$$|\Psi(\Sigma_{out})\rangle = U|\Psi(\Sigma_{in})\rangle, \quad |\bar{\Psi}(\Sigma_{out})\rangle = U|\bar{\Psi}(\Sigma_{in})\rangle, \tag{7}$$

$$|\bar{\Psi}(\Sigma_{in})\rangle = U_0|\Psi(\Sigma_{in})\rangle, \quad |\bar{\Psi}(\Sigma_{out})\rangle = U_0|\Psi(\Sigma_{out})\rangle. \tag{8}$$

Due to this separation, the calculated tripartite mutual information can be separated into two contributions. One is for gravity, and the other is for other interaction, in our case, QED.

Tripartite mutual information in collapsing space-time

The formal expression of the tripartite mutual information $I_{3(2)}$ is

$$\begin{aligned} I_{3(2)} &= I^0_{3(2)} + I^1_{3(2)} + \cdots \\ &= -\frac{4}{D} \sum_{\{1^{e^-}_Q\}} \sum_{\{Q\}} \sum_{\{\zeta^l_a\}} \sum_{\{1^{e^-}_G\}} \sum_{\{G\}} \sum_{\{\alpha^l_a\}} |_{in}\langle 1^{e^-}_G, \{\alpha^l_a\}|G_0|1^{e^-}_Q, \{\zeta^l_a\}\rangle_{in}|^2 \\ &\quad -\frac{4}{D} \sum_{\{1^{e^-}_Q\}} \sum_{\{Q\}} \sum_{\{\zeta^l_a\}} \sum_{\{1^{e^-}_G\}} \sum_{\{G\}} \sum_{\{\alpha^l_a\}} |_{in}\langle 1^{e^-}_G, \{\alpha^l_a\}|T|1^{e^-}_Q, \{\zeta^l_a\}\rangle_{in}|^2 + \cdots \\ &\simeq (I^0_{3(2)})^{detectable} + (I^0_{3(2)})^{undetectable} + (I^1_{3(2)})^{detectable}. \end{aligned} \tag{9}$$

The separation of $(I^0_{3(2)})^{detectable}$ and $(I^0_{3(2)})^{undetectable}$ is related to the fact that the singularity of the black hole divides the space-time into two regions.

Tripartite mutual information in collapsing space-time

Note: The states $|\{\beta_a^l\}\rangle_{in}$ label the photon coherent states (at the end of calculation, the convergence condition is needed for correct soft dressing). They can expressed as

$$|\{\beta_a^l\}\rangle_{in} = \prod_{l,a} |\beta_a^l\rangle_{in}, \tag{10}$$

and

$$|\beta_a^l\rangle_{in} = \exp\Big[\beta_a^l (c_a^{l\dagger})_{in} - \beta_a^{l*} (c_a^l)_{in}\Big] |0\rangle_{in} = \exp\Big(-\frac{1}{2}|\beta_a^l|^2\Big) \exp\Big(\beta_a^l (c_a^{l\dagger})_{in}\Big) |0\rangle_{in}, \tag{11}$$

where

$$(c_a^{l\dagger})_{in} = \int d\{q\} f_a(q) c_{in}^{(l)\dagger}(q). \tag{12}$$

l denotes the photon polarization of the photon state, $\{q\}$ denotes other quantum numbers of the photon state, $f_a(q)$ is a set of distribution functions for the relevant quantum numbers except polarization.

Tripartite mutual information in collapsing space-time

Define $(I_{3(2)})^{detectable}$ as

$$(I_{3(2)})^{detectable} \equiv (I^0_{3(2)})^{detectable} + (I^1_{3(2)})^{detectable} \geq I_{3(2)}. \tag{13}$$

After calculation, one will find

$$\begin{aligned} I^{\rm grav}_{3(2)} = (I^0_{3(2)})^{detectable} &\simeq -\frac{4C}{(8\pi)^6}\Big(\frac{\delta M m}{M_{pl}^2}\Big)^6/\Phi_0 \simeq -(10^{-8})\Big(\frac{\delta M m}{M_{pl}^2}\Big)^6/\Phi_0 \\ &\gg -(10^{-8})/\Phi_0, \end{aligned} \tag{14}$$

$$\begin{aligned} I^{\rm QED}_{3(2)} = (I^1_{3(2)})^{detectable} &\simeq -8\pi^2(2\pi)^5 A_1(\alpha Z)^2/\Phi_0 \simeq -(10^5)(\alpha Z)^2/\Phi_0 \\ &\simeq -(10^1)/\Phi_0. \end{aligned} \tag{15}$$

Φ_0 is the normalized flux.

Discussion

- After including soft degrees of freedom, we can see that the information encoded in hard particles of the input states is scrambled during the unitary evolution.
- The detectable gravity contribution of the tripartite mutual information is much smaller than QED contribution of the tripartite mutual information.
- Our study provides a new perspective on the problem of the black hole information paradox. That is, when soft degrees of freedom are included, the interaction naturally scrambles the information.

The End