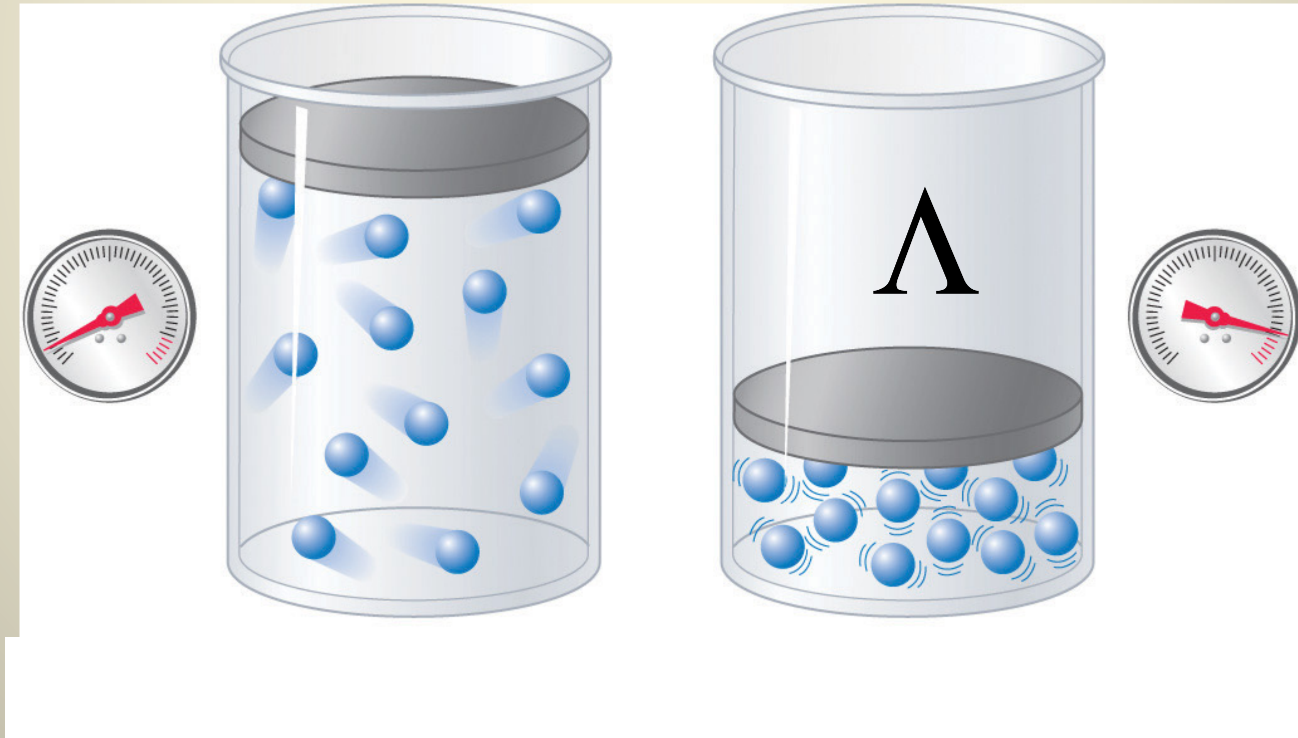
BLACK HOLE CHEMISTRY

Robert Mann

Black Hole Chemistry: Pressure from the Vacuum

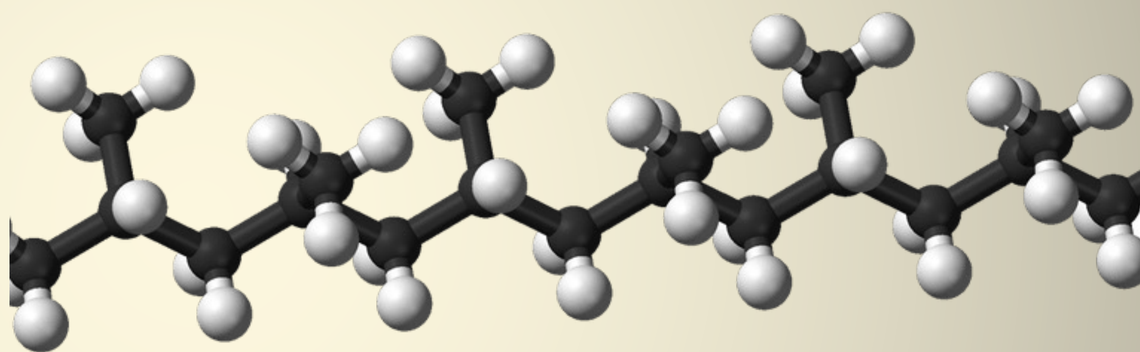
$$p = -\frac{\Lambda}{8\pi G}$$



Intermediate Results from Black Hole Chemistry

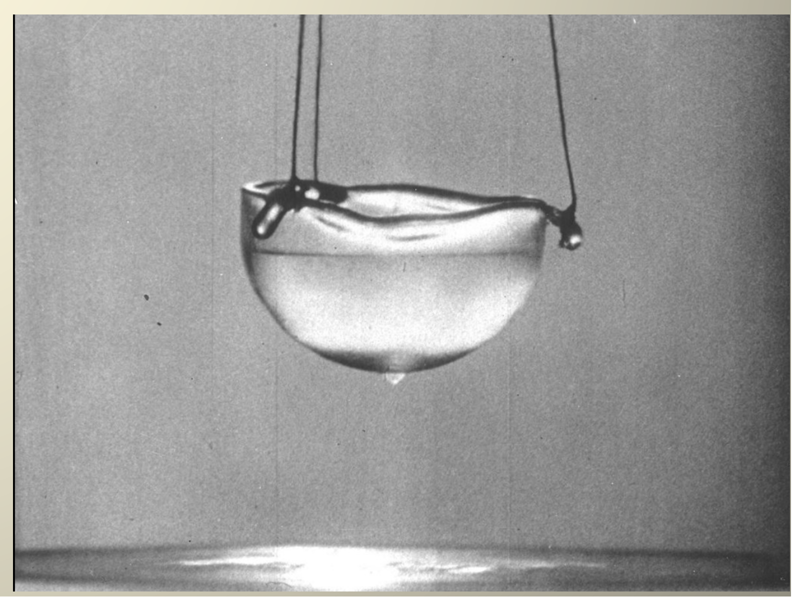
- Polymer Black holes
  – Some black holes have polymer transitions with non-standard critical exponents

B. Dolan, A. Kostouki, D.Kubiznak, R.B. Mann, CQG **31** (2014) 242001



- Superfluid Black Holes
  – Black holes with scalar “hair” can exhibit a superfluid phase transition analogous to ^{4}He

E. Tjoa, R. Hennigar, R.B. Mann PRL **118** (2017) 021301
E. Tjoa, R. Hennigar, R.B. Mann JHEP **1702** (2017) 040
H. Dykaar, R. Hennigar, R.B. Mann JHEP **1705** (2017) 045



Johnson CQG **31** (2014) 2005002
Henningar/McCarthy/Ballon/Mann
CQG **34** (2017) 175005

- Black Hole Heat Engines
  - Can benchmark various black holes for their efficiencies as heat engines → maximize efficiency

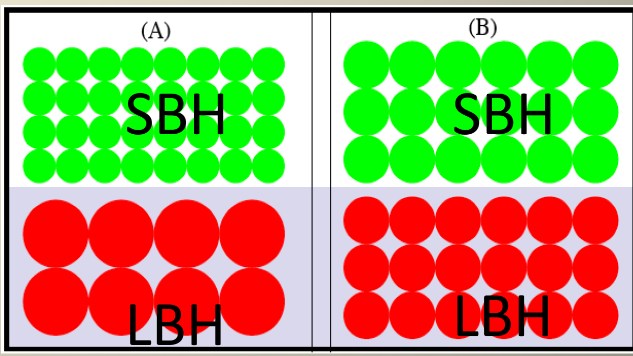
$$\eta \leq \eta_{\circ} = \frac{2\pi}{\pi+4}$$

Theory-independent upper limit on BH efficiency!

- De Sitter Black Hole Chemistry
  - Cosmic Volume
  - Swallowtubes

Dolan/Kastor/Kubiznak/Mann/Traschen
PRD 87 (2013) 104017
Mbarek/Mann PLB 765 (2017) 352
F. Simovic and R.B. Mann
CQG 36 (2019) 014002;
JHEP 1905 (2019) 136

- Black Hole Molecules
  - Microstructure appears to be molecular
  - Small charged AdS BHs have repulsive microstructure interactions



Wei /Liu PRL **115** (2015) 11132
Wei /Liu /Mann PRL **123** (2019) 071103

Polymeric Black Holes

Lovelock Gravity

$$\mathcal{L} = \frac{1}{16\pi}\sum_{k=0}^{K}\alpha_k \mathcal{L}^{(k)} + L_m$$

$$\mathcal{L}^{(k)} = \frac{1}{2^k}\delta^{a_1 b_1 \dots a_k b_k}_{c_1 d_1 \dots c_k d_k} R_{a_1 b_1}{}^{c_1 d_1} \dots R_{a_k b_k}{}^{c_k d_k}$$

$$= \frac{1}{16\pi}\left(\alpha_0 + \alpha_1 R + \alpha_2\left(R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2\right) + \cdots\right)$$

$$S = \frac{1}{16\pi}\int d^D x\sqrt{-g}\left(\sum_{k=0}^{K}\alpha_k \mathcal{L}^{(k)} + L_m\right)$$

$$K \le \frac{D-1}{2}$$

Special Cases

Chern-Simons Gravity

$$\hat{\alpha}_p = \frac{\ell^{2p-2n+1}}{2n-2p-1}\binom{n-1}{p}$$

$$p = 1,2,\dots,n-1 = \frac{d-1}{2}$$

Isolated Critical Point

$$\hat{\alpha}_k = \hat{\alpha}_K (K\hat{\alpha}_K)^{-\frac{K-k}{K-1}}\binom{K}{k}$$

$$\text{for} \quad 2 \le k < K,$$

Miskovic/Zanelli PRD **80** (2009) 044003

Equations of Motion

$$S=\frac{1}{16\pi}\int d^Dx\sqrt{-g}\left(\sum_{k=0}^{K}\alpha_k\mathcal{L}^{(k)}-\frac{1}{4}F^{ab}F_{ab}\right)$$

$$\mathcal{G}^a_b=\sum_{k=0}^{K}\alpha_k\mathcal{G}^{(k)a}{}_b=8\pi T^a{}_b$$

$$\mathcal{G}^{(k)a}{}_b=-\frac{1}{2^{(k+1)}}\delta^{ac_1d_1\ldots c_kd_k}_{be_1f_1\ldots e_kf_k}R_{c_1d_1}{}^{e_1f_1}\ldots R_{c_kd_k}{}^{e_kf_k}$$

$$\hat{\alpha}_0=\frac{\alpha_0}{(D-1)(D-2)}\qquad\hat{\alpha}_1=\alpha_1$$

$$\hat{\alpha}_k=\alpha_k\prod_{n=3}^{2k}(D-n)\quad\text{for}\quad k\geq 2$$

Spherical Symmetry

$$ds^2=-f(r)dt^2+f(r)^{-1}dr^2+r^2d\Omega^2_{(\kappa)d-2}\qquad F=\frac{Q}{r^{D-2}}dt\wedge dr\qquad\kappa=-1,0,1$$

Field Equation

Lovelock Polynomial

$$P(f)=\sum_{k=0}^{K}\hat{\alpha}_k\left(\frac{\kappa-f}{r^2}\right)^k=\frac{16\pi G_NM}{(D-2)\Sigma^{(\kappa)}_{D-2}r^{D-1}}-\frac{8\pi G_NQ^2}{(D-2)(D-3)}\frac{1}{r^{2D-4}}=X(r)$$

Solution to the Lovelock Polynomial equation gives the metric function

- In general there are K branches $\rightarrow$ one gives Einstein gravity as $\alpha_{k\geq 2}\rightarrow 0$

Gauss-Bonnet Gravity

$$f=\frac{\hat{\alpha}_1r^2+2\kappa\hat{\alpha}_2\pm\sqrt{4X(r)\hat{\alpha}_2r^4-4\hat{\alpha}_0\hat{\alpha}_2r^4+\hat{\alpha}_1^2r^4}}{\hat{\alpha}_2}$$

Einstein branch

First Law and Smarr Formula

$$S = \frac{1}{16\pi}\int d^D x\sqrt{-g}\left(\sum_{k=0}^{K}\alpha_k \mathcal{L}^{(k)} - 4b^2\left(1-\sqrt{1+\frac{2F^2}{b^2}-\frac{\tilde{F}^2}{b^4}}\right)\right)$$

Lovelock Gravity

Non-linear Electrodynamics

$$F^2 = \frac{1}{4}F^{ab}F_{ab}$$

$$\tilde{F}^2 = \frac{1}{8}\epsilon^{abcd}F_{ab}F_{cd}$$

Same Geometric arguments as before

Kastor/Ray/Traschen
CQG 27 (2010) 235014

$$\delta M = T\delta S + V\delta P + \sum_{k=2}^{K}\Psi^k \delta\alpha_k + \sum_i \Omega^i \delta J^i + \sum_j \Phi^j \delta Q^j + Bdb$$

1st Law

Also follows from Eulerian scaling

$$\frac{D-3}{D-2}M = TS - \frac{2}{D-2}PV + \sum_{k=2}^{K}\frac{2(k-1)}{D-2}\Psi^k\alpha_k + \sum_i \Omega^i J^i + \frac{D-3}{D-2}\sum_j \Phi^j Q^j - \frac{1}{D-2}Bb$$

Smarr

Gunasekaran/Kubiznak/Mann
JHEP 1211 (2012) 110

Be careful!

Entropy $$S = \frac{1}{4G_N}\sum_k \alpha_k \mathcal{A}^{(k)}$$ → Not always positive

Lovelock Potentials $$\hat{\Psi}^{(k)} = 4\pi T \mathcal{A}^{(k)} + \mathcal{B}^{(k)} + \Theta^{(k)}$$

$$\mathcal{A}^{(k)} = k\int_{\mathcal{H}}\sqrt{\sigma}\,\mathcal{L}^{(k-1)}$$

$$\mathcal{B}^{(k)} = -\frac{16\pi k G_N M (D-1)!}{b(D-2k-1)!}\left(-\frac{1}{\ell^2}\right)^{k-1} \qquad b = \sum_k \frac{\alpha_k k (D-1)!}{(D-2k-1)!}\left(-\frac{1}{\ell^2}\right)^{k-1}$$

$$\Theta^{(k)} = \int_{\Sigma}\sqrt{-g}\,\mathcal{L}^{(k)}[s] - \int_{\Sigma_{\mathrm{AdS}}}\sqrt{-g_{\mathrm{AdS}}}\,\mathcal{L}^{(k)}[s_{\mathrm{AdS}}]$$

Back to Spherical/Planar/Hyperbolic Geometry

$\kappa = -1, 0, 1$

$$P(f) = \sum_{k=0}^{K} \hat{\alpha}_k \left(\frac{\kappa - f}{r^2} \right)^k = \frac{16\pi G_N M}{(D-2)\Sigma_{D-2}^{(\kappa)} r^{D-1}} - \frac{8\pi G_N Q^2}{(D-2)(D-3)} \frac{1}{r^{2D-4}}$$

Thermodynamic Quantities

$$\Sigma_{d-2}^{(+1)} = \frac{2\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)}$$

$$M = \frac{\Sigma_{D-2}^{(\kappa)}(D-2)}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_k \kappa^k r_+^{D-1-2k} + \frac{\Sigma_{D-2}^{(\kappa)}}{2(d-3)} \frac{Q^2}{r_+^{D-3}}$$

$$D(r_+) = \sum_{k=1}^{K} k \hat{\alpha}_k \left(\kappa r_+^{-2}\right)^{k-1}$$

$$T = \frac{|f'(r_+)|}{4\pi} = \frac{1}{4\pi r_+ D(r_+)} \left[\sum_k \kappa \hat{\alpha}_k (D-2k-1) \left(\frac{\kappa}{r_+^2}\right)^{k-1} - \frac{8\pi G_N Q^2}{(D-2) r_+^{2(D-3)}} \right]$$

$$S = \frac{\Sigma_{D-2}^{(\kappa)}(D-2)}{4G_N} \sum_{k=0}^{K} \frac{k \kappa^{k-1} \alpha_k r_+^{D-2k}}{D-2k} \qquad \Phi = \frac{\Sigma_{D-2}^{(\kappa)} Q}{(D-3) r_+^{d-3}}$$

Frassino/Kubiznak/Mann/Simovic JHEP 1409 (2014) 080

$$\Psi^{(k)} = \frac{\Sigma_{D-2}^{(\kappa)}(D-2)}{16\pi G_N} \kappa^{k-1} r_+^{D-2k} \left[\frac{\kappa}{r} - \frac{4\pi k T}{D-2k} \right] \qquad V = \frac{16\pi G_N \Psi^{(0)}}{(D-1)(D-2)} = \frac{\Sigma_{D-2}^{(\kappa)} r_+^{D-1}}{D-1}$$

Equation of State

$$P = P(V, T, Q, \alpha_1, \ldots, \alpha_{k_{max}}) = \frac{D-2}{16\pi G_N} \sum_{k=1}^{K} \frac{\hat{\alpha}_k}{r_+^2} \left(\frac{\kappa}{r_+^2}\right)^{k-1} [4\pi k r_+ T - \kappa(D-2k-1)] + \frac{Q^2}{2\alpha_1 r_+^{2(D-2)}}$$

3^{rd} Order Lovelock Thermodynamics

$$P = P(V,T,Q,\alpha_1,\ldots,\alpha_{k_{max}}) = \frac{D-2}{16\pi G_N}\sum_{k=1}^{K}\frac{\hat{\alpha}_k}{r_+^2}\left(\frac{\kappa}{r_+^2}\right)^{k-1}[4\pi k r_+ T - \kappa(D-2k-1)] + \frac{Q^2}{2\alpha_1 r_+^{2(D-2)}}$$

$$r_+ = v\alpha_3^{1/4} \quad T = \frac{t\alpha_3^{-1/4}}{d-2} \quad m = \frac{16\pi M}{(D-2)\Sigma_{D-2}^{(\kappa)}\alpha_3^{\frac{D-3}{4}}} \quad Q = \frac{q}{\sqrt{2}}\alpha_3^{\frac{D-3}{4}} \quad \alpha = \frac{\alpha_2}{\sqrt{\alpha_3}}$$

Example: $D = 7$ $K = 3$

case	range of α	# critical points	behavior
I	$\alpha \in (0, \sqrt{5/3})$	1	VdW
II	$\alpha \in (\sqrt{5/3}, \sqrt{3})$	2	VdW & reverse VdW
III	$\alpha = \sqrt{3}$	1	special
IV	$\alpha \in (\sqrt{3}, 3\sqrt{3/5})$	0	infinite coexistence line
V	$\alpha > 3\sqrt{3/5}$	0	multiple RPT, infinite coexistence line

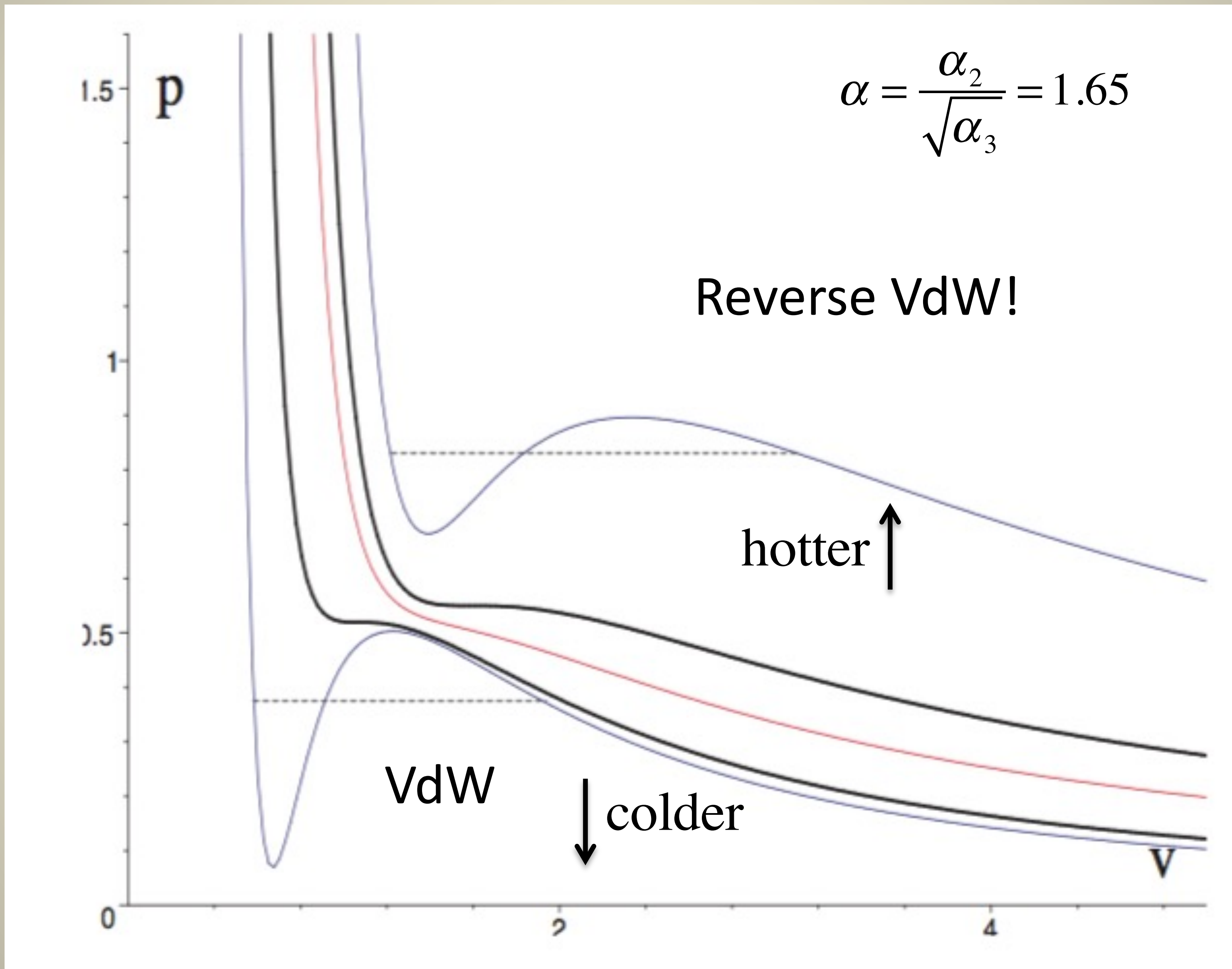
Reverse VdW Behaviour

$D = 7$

$K = 3$

$\kappa = -1$

$q = 0$

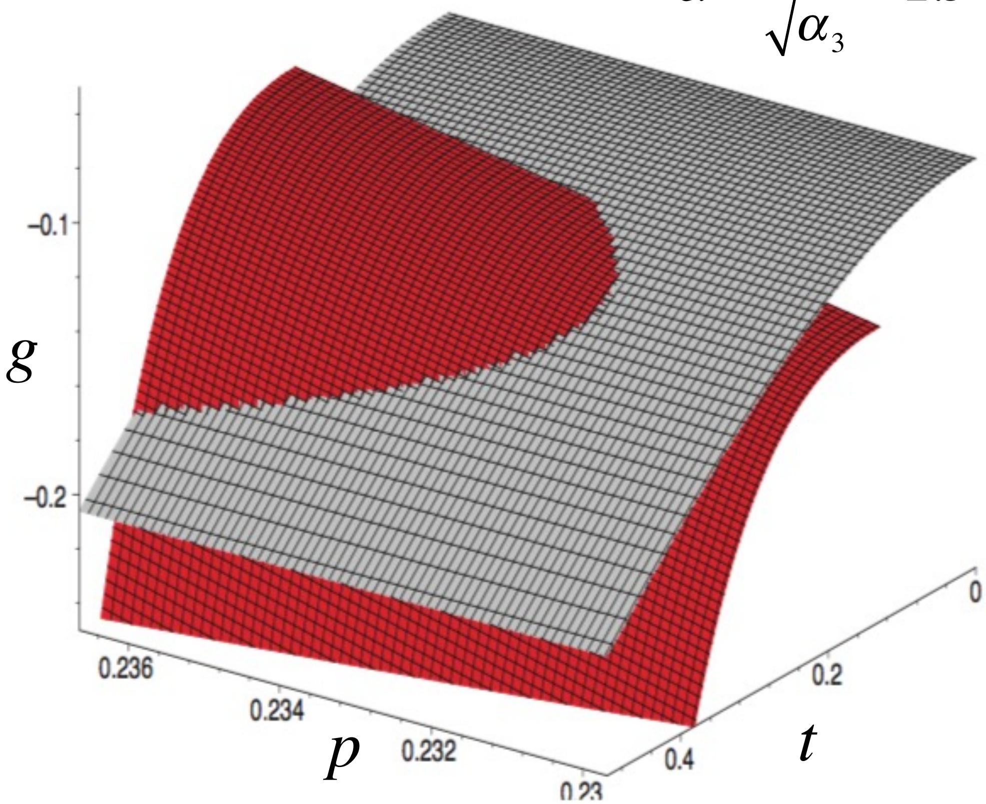


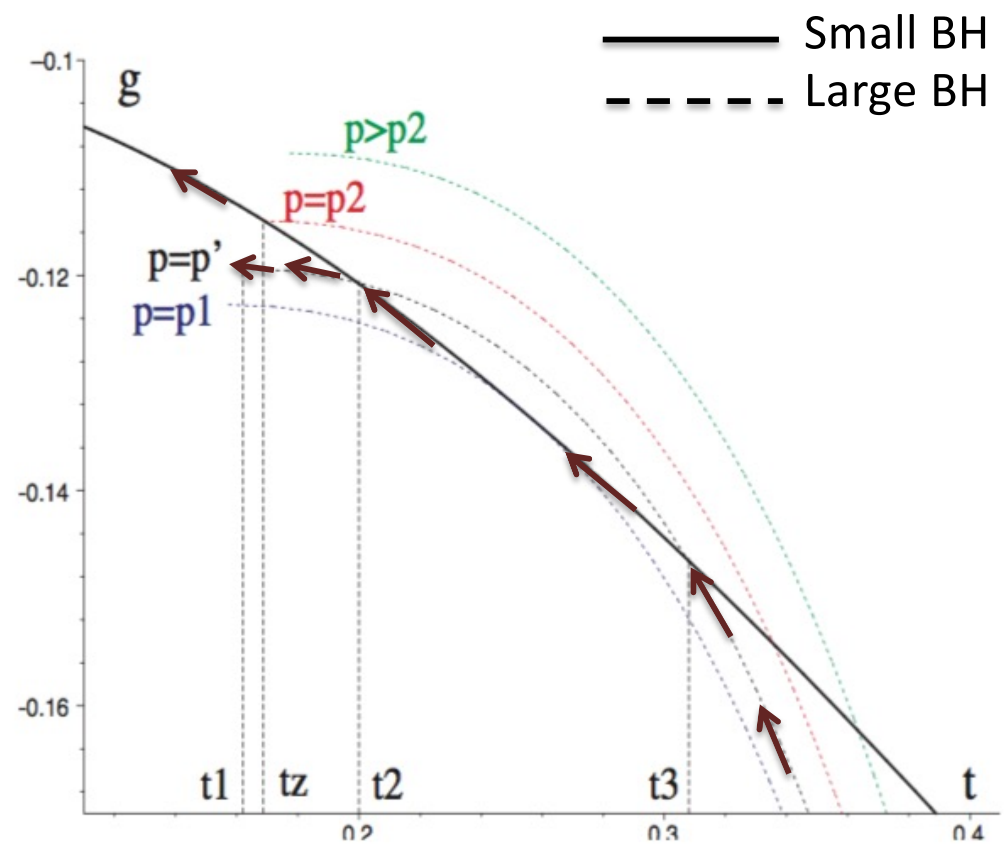
Multiple Re-Entrant Phase Transitions

$D = 7 \quad K = 3$

$\kappa = \text{-}1 \quad q = 0$

$\alpha = \frac{\alpha_2}{\sqrt{\alpha_3}} = 2.5$





Isolated Critical Points

B. Dolan, A. Kostouki,
D.Kubiznak,
R.B. Mann,
CQG 31 (2014) 242001

$$\mathcal{L} = \frac{1}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_{(k)} \mathcal{L}^{(k)} = \frac{2^{-k}}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_{(k)} \delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R_{a_1 b_1}{}^{c_1 d_1} \ldots R_{a_k b_k}{}^{c_k d_k}$$

$$\hat{\alpha}_k = \alpha A^{K-k} \begin{pmatrix} K \\ k \end{pmatrix}$$

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_\kappa^2$$

$$A^{-1} = \sqrt[K-1]{K\alpha}$$

$$f = \kappa + r^2 A \left[1 - \left(\frac{m_0 r^{1-D} - \alpha_0}{\alpha A^K} + 1 \right)^{1/K} \right]$$

$$B \equiv \frac{\kappa}{r_+^2} + A$$

Equation of State

$$P = \frac{(D-1)(D-2)\alpha}{16\pi G_N} \left[B^{K-1} \left(\frac{2K(2\pi r_+ T + \kappa)}{(D-1) r_+^2} - B \right) + A^K \right]$$

$$\kappa = -1$$

$$K = \text{odd}$$

Isolated Critical Point where two 1st order Phase Transitions merge

$$\kappa = -1 \qquad \frac{\partial^k P}{\partial r_+^k} = 0$$

$$K = \text{odd} \qquad k = 1, \ldots, K-1$$

$$r_c = \frac{1}{\sqrt{A}} \qquad T_c = \frac{1}{2\pi r_c}$$

$$P_c = \frac{(D-1)(D-2)\alpha}{16\pi G_N} A^K$$

$$\frac{P}{P_c} = 1 + K\frac{2^K}{D-1}\omega^{K-1}\tau + \frac{(K-D+1)2^K}{D-1}\omega^K + \ldots$$

$$\tilde{\alpha} = 0, \; \tilde{\beta} = 1, \; \tilde{\gamma} = K-1, \; \tilde{\delta} = K$$

Non-Standard Critical Exponents

$$\frac{p}{p_c} = 1 + A\frac{t}{t_c} + B\frac{t}{t_c}\frac{v-v_c}{v_c} + C\left(\frac{v-v_c}{v_c}\right)^3 + \ldots$$

Most black holes ←→Mean field thy

→standard critical exponents

$$\frac{p}{p_c} = 1 + B\frac{t}{t_c}\left(\frac{v-v_c}{v_c}\right)^2 + C\left(\frac{v-v_c}{v_c}\right)^3 + .$$

Topological black holes ←→scaling rel'ns violated

→non-standard critical exponents

Note: Isolated critical points do not require either thermodynamic singularities or hyperbolic black holes!

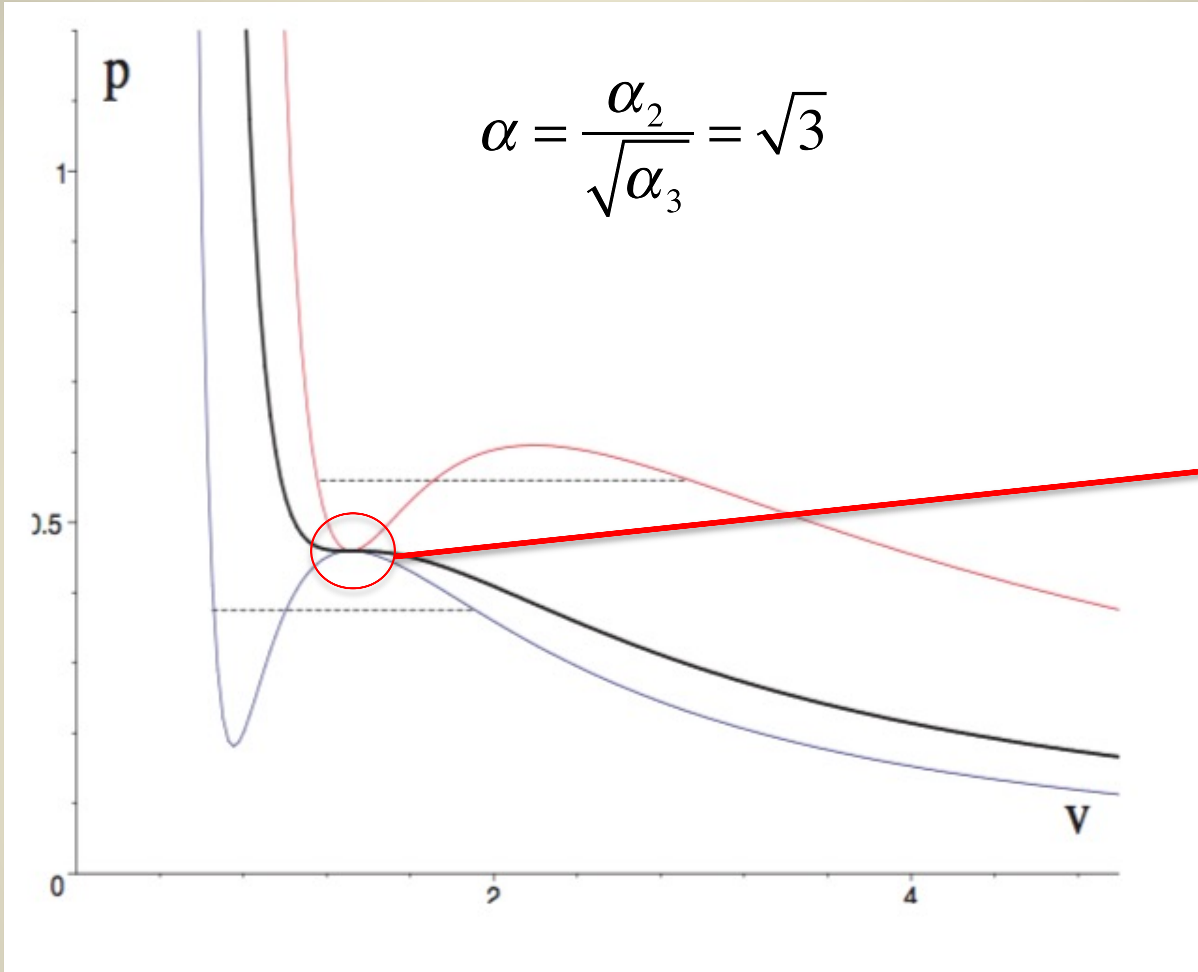
Dykaar/Hennigar/Mann

JHEP 1705 (2017) 045

Isolated Critical Point

$D = 7 \quad K = 3$

$\kappa = -1 \quad q = 0$



Standard

$\tilde{\alpha} = 0$

$\tilde{\beta} = 1/2$

$\tilde{\gamma} = 1$

$\tilde{\delta} = 3$

Non-Standard

$\tilde{\alpha} = 0$

$\tilde{\beta} = 1$

$\tilde{\gamma} = 2$

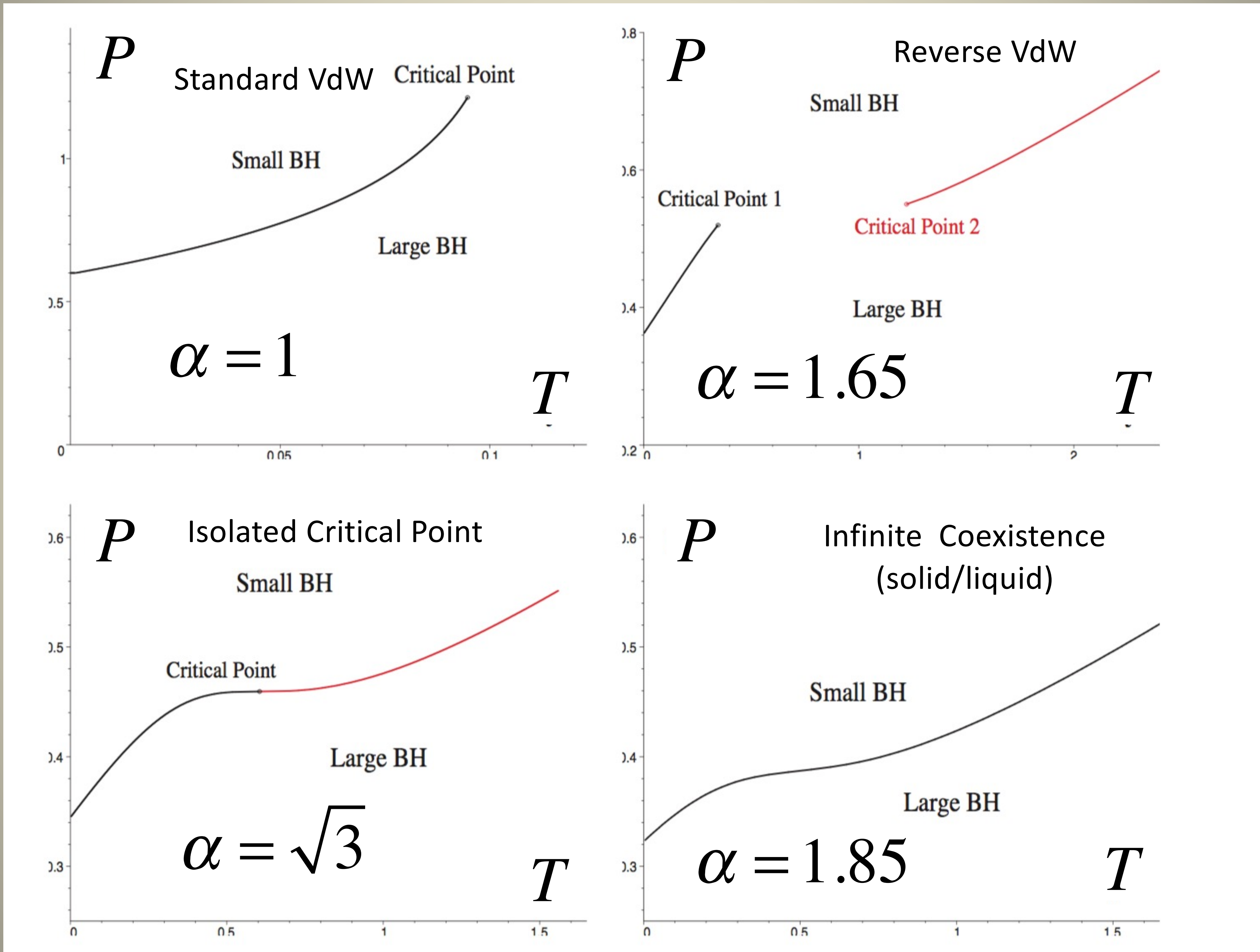
$\tilde{\delta} = 3$

$$\frac{p}{p_c} = 1 + A\frac{t}{t_c} + B\frac{t}{t_c}\frac{v - v_c}{v_c} + C\left(\frac{v - v_c}{v_c}\right)^3 + \ldots$$

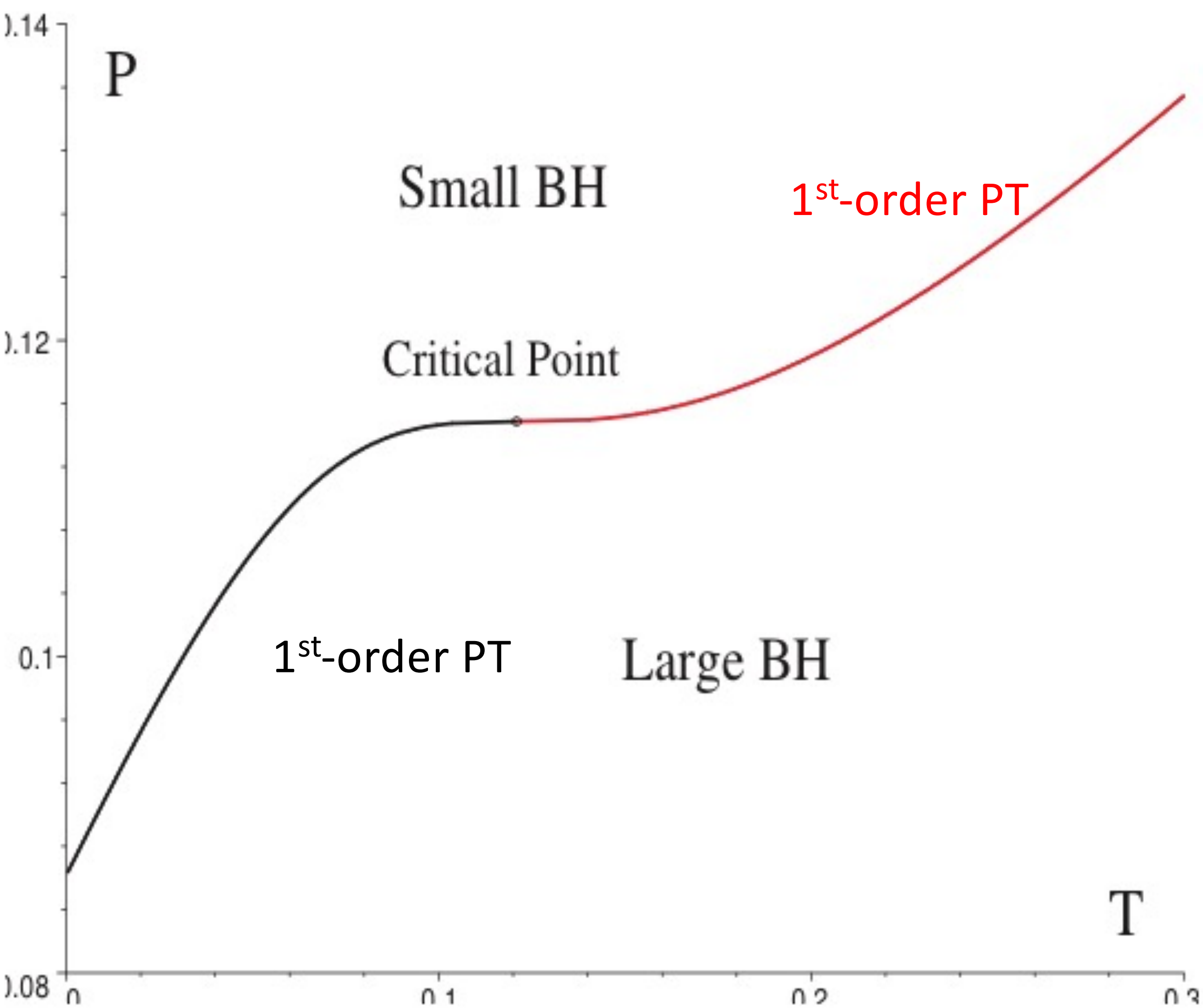
Most black holes ←→Mean field thy
→standard critical exponents

$$\frac{p}{p_c} = 1 + B\frac{t}{t_c}\left(\frac{v - v_c}{v_c}\right)^2 + C\left(\frac{v - v_c}{v_c}\right)^3 + \ldots$$

Topological black holes ←→scaling rel'ns violated
→non-standard critical exponents


P
Standard VdW
Critical Point
Small BH
Large BH
α = 1
T
P
Reverse VdW
Small BH
Critical Point 1
Critical Point 2
Large BH
α = 1.65
T
P
Isolated Critical Point
Small BH
Critical Point
Large BH
α = √3
T
P
Infinite Coexistence
(solid/liquid)
Small BH
Large BH
α = 1.85
T

$$\tilde{\alpha} = 0, \ \tilde{\beta} = 1, \ \tilde{\gamma} = K - 1, \ \tilde{\delta} = K$$



$$\tilde{\gamma} = \tilde{\beta}(\tilde{\delta} - 1)$$

Widom Relation ✓

$$\tilde{\alpha} + 2\tilde{\beta} + \tilde{\gamma} \geq 2$$

Rushbrooke inequality ✓

Ehrenfest relations ✓

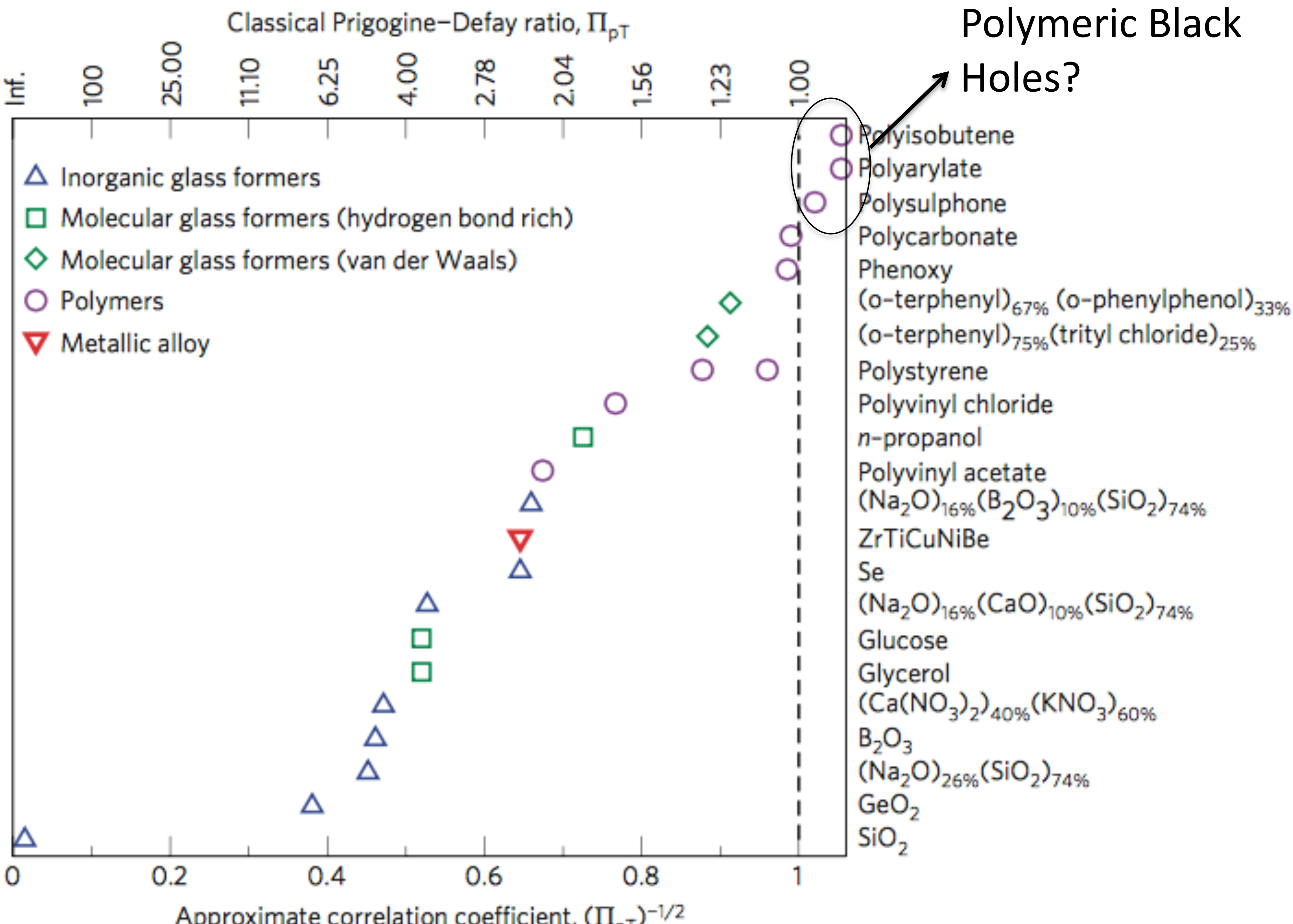
$$\Pi = 1/K$$

Prigogine-De Fay ratio

Suggests a liquid-glass type of phase transition

Similar phenomena present for Quasi-topological Black Holes

W.G. Brenna, R. Hennigar, R.B. Mann, JHEP 1507 (2015) 077

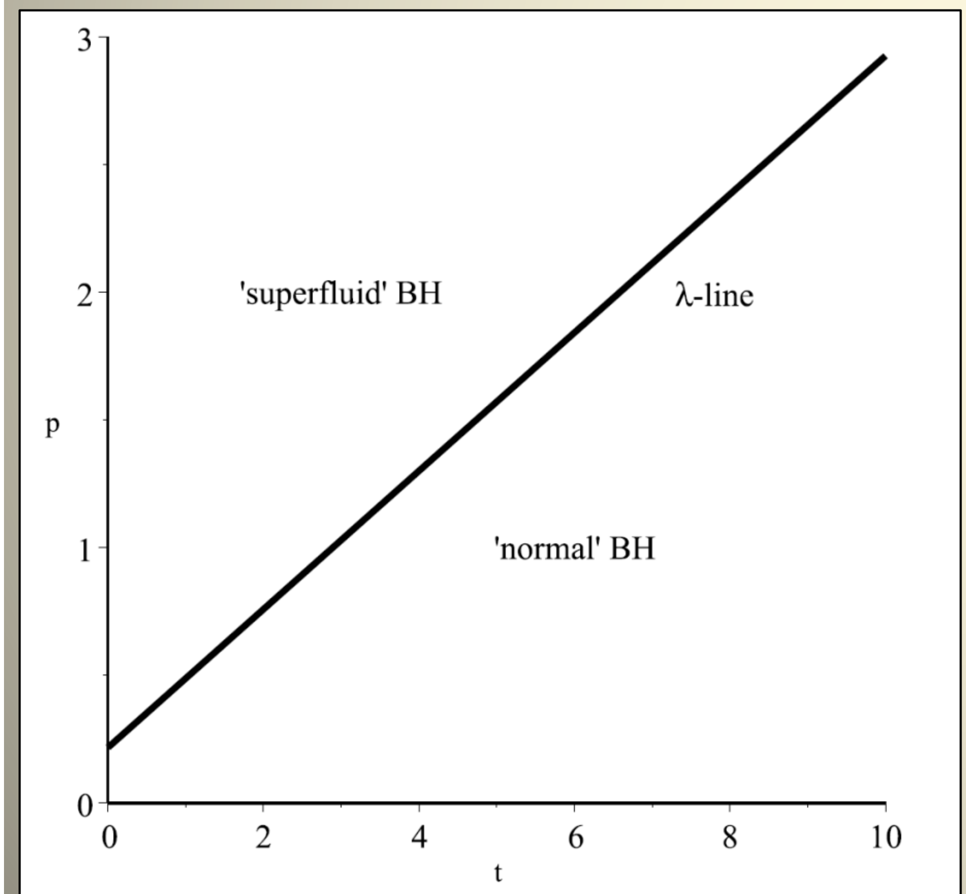


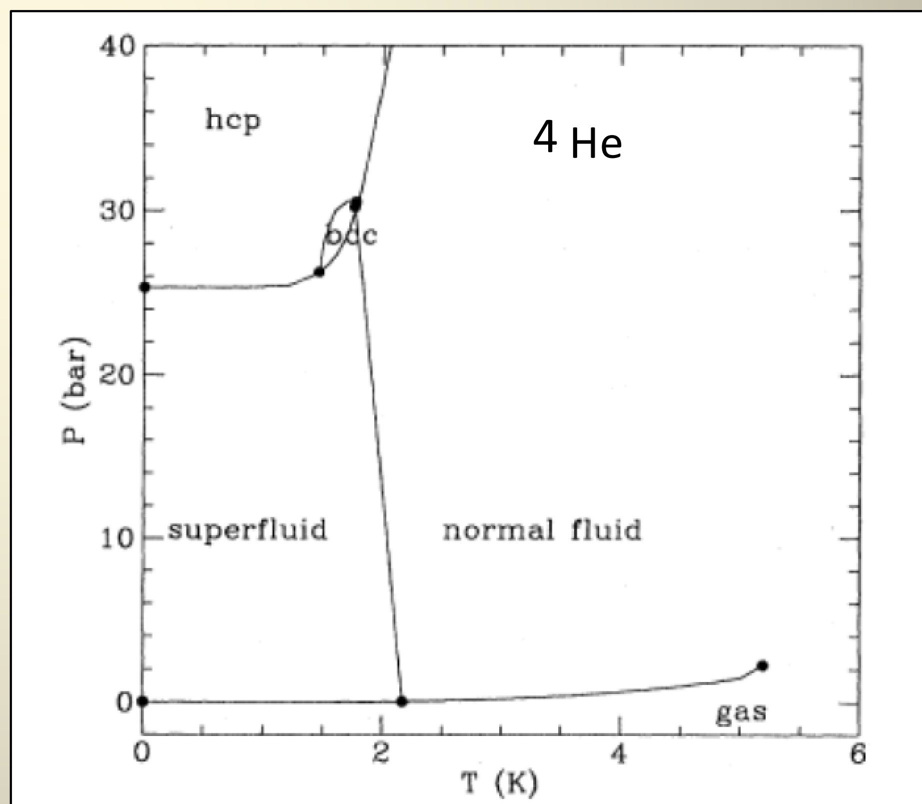
Classical Prigogine–Defay ratio, Π_{pT}
Inf.
100
25.00
11.10
6.25
4.00
2.78
2.04
1.56
1.23
1.00
Polymeric Black Holes?
Inorganic glass formers
Molecular glass formers (hydrogen bond rich)
Molecular glass formers (van der Waals)
Polymers
Metallic alloy
Polyisobutene
Polyarylate
Polysulphone
Polycarbonate
Phenoxy
(o-terphenyl)$_{67\%}$ (o-phenylphenol)$_{33\%}$
(o-terphenyl)$_{75\%}$(trityl chloride)$_{25\%}$
Polystyrene
Polyvinyl chloride
n-propanol
Polyvinyl acetate
$(Na_2O)_{16\%}(B_2O_3)_{10\%}(SiO_2)_{74\%}$
ZrTiCuNiBe
Se
$(Na_2O)_{16\%}(CaO)_{10\%}(SiO_2)_{74\%}$
Glucose
Glycerol
$(Ca(NO_3)_2)_{40\%}(KNO_3)_{60\%}$
B_2O_3
$(Na_2O)_{26\%}(SiO_2)_{74\%}$
GeO_2
SiO_2
0
0.2
0.4
0.6
0.8
1
Approximate correlation coefficient, $(\Pi_{pT})^{-1/2}$

Superfluid Black Holes

E. Tjoa, R. Hennigar, R.B. Mann,
PRL **118** (2017) 021301
JHEP **1702** (2017) 070

- A line of continuous phase transitions
- Reminiscent of the ^{4}He transition marking the onset of superfluidity
- Observed for hairy black holes in higher dimensions
- Now known to occur in a broad class of higher-curvature theories





Conformally Hairy Black Holes

$$S=\frac{1}{16\pi}\int d^Dx\sqrt{-g}\left(\sum_{k=0}^{K}\alpha_k\mathcal{L}^{(k)}-\frac{1}{4}F^{ab}F_{ab}\right)$$

$$\hat{\alpha}_0=\frac{\alpha_0}{(D-1)(D-2)}\quad \hat{\alpha}_1=\alpha_1$$

$$\hat{\alpha}_k=\alpha_k\prod_{n=3}^{2k}(D-n)\quad \text{for}\quad k\geq 2$$

$$\mathcal{L}^{(k)}=\frac{1}{2^k}\delta^{(k)}\left(\alpha_k\prod_r^k R^{\alpha_r\beta_r}_{\mu_r\nu_r}+\beta_k\phi^{d-4k}\prod_r^k S^{\alpha_r\beta_r}_{\mu_r\nu_r}\right)$$

$$\delta^{(k)}=\delta^{a\,c_1d_1\ldots c_kd_k}_{b\,e_1f_1\ldots e_kf_k}$$

$$\mathcal{L}^{(k)}=\frac{1}{2^k}\delta^{a_1b_1\ldots a_kb_k}_{c_1d_1\ldots c_kd_k}R_{a_1b_1}{}^{c_1d_1}\ldots R_{a_kb_k}{}^{c_kd_k}$$

J. Oliva and S. Ray
CQG **29** (2012) 205008,
G. Giribet, M. Leoni, J. Oliva, and S. Ray
Phys. Rev. **D89** (2014), 085040

$$S_{\mu\nu}{}^{\gamma\delta}=\phi^2\ R_{\mu\nu}{}^{\gamma\delta}-2\delta^{[\gamma}_{[\mu}\delta^{\delta]}_{\nu]}\nabla_\rho\phi\nabla^\rho\phi-4\phi\delta^{[\gamma}_{[\mu}\nabla_{\nu]}\nabla^{\delta]}\phi+8\delta^{[\gamma}_{[\mu}\nabla_{\nu]}\phi\nabla^{\delta]}\phi$$

Spherical Symmetry

$$ds^2=-f(r)dt^2+f(r)^{-1}dr^2+r^2d\Omega^2_{(\kappa)d-2}$$

$$\kappa=-1,0,1\qquad F=\frac{Q}{r^{D-2}}dt\wedge dr$$

Field Equation

$$\phi=\frac{N}{r}\qquad H=\sum_{k=0}^{K}\frac{(D-3)!}{(D-2(k+1))!}\beta_k\kappa^kN^{D-2k}$$

$$P(f)=\sum_{k=0}^{K}\hat{\alpha}_k\left(\frac{\kappa-f}{r^2}\right)^k=\frac{16\pi G_NM}{(D-2)\Sigma^{(\kappa)}_{D-2}r^{D-1}}-\frac{8\pi G_NQ^2}{(D-2)(D-3)}\frac{1}{r^{2D-4}}+\frac{H}{r^d}$$

$$P(f)=\sum_{k=0}^{K}\hat{\alpha}_k\left(\frac{\kappa-f}{r^2}\right)^k=\frac{16\pi G_N M}{(D-2)\Sigma_{D-2}^{(\kappa)}r^{D-1}}-\frac{8\pi G_N Q^2}{(D-2)(D-3)}\frac{1}{r^{2D-4}}+\frac{H}{r^d}$$

$$\sum_{k=1}^{K}k\beta_k\frac{(D-1)!}{(D-2k-1)!}\kappa^{k-1}N^{2-2k}=0 \qquad \phi=\frac{N}{r} \qquad H=\sum_{k=0}^{K}\frac{(D-3)!}{(D-2(k+1))!}\beta_k\kappa^k N^{D-2k}$$

$$\sum_{k=0}^{K}\beta_k\frac{(D-1)!\left(D(D-1)+4k^2\right)}{(D-2k-1)!}\kappa^k N^{-2k}=0$$

$$M=\frac{(D-2)\Sigma_{D-2}^{\kappa}}{16\pi G}\sum_{k=0}^{K}\alpha_k\kappa^k r_+^{D-2k-1}-\frac{(D-2)\Sigma_{D-2}^{\kappa}H}{16\pi G r_+}+\frac{\Sigma_{D-2}^{\kappa}Q^2}{2(D-3)r_+^{D-3}}$$

$$T=\frac{1}{4\pi r_+ D(r_+)}\left[\kappa\alpha_k(D-2k-1)\left(\frac{\kappa}{r_+^2}\right)^{k-1}+\frac{H}{r_+^{D-2}}-\frac{8\pi G Q^2}{(D-2)r_+^{2(d-3)}}\right]$$

$$S=\frac{\Sigma_{D-2}^{\kappa}}{4G}\left[\sum_{k=1}^{k_{\max}}\frac{(D-2)k\kappa^{k-1}\alpha_k}{D-2k}r_+^{d-2k}-\frac{D}{2\kappa(D-4)}H\right] \qquad D(r_+)=\sum_{k=1}^{K}k\alpha_k(\kappa r_+^{-2})^{k-1}$$

$$r_+=v\alpha_3^{1/4} \quad T=\frac{t\alpha_3^{-1/4}}{D-2} \quad H=\frac{4\pi h}{D-2}\alpha_3^{\frac{D-2}{4}} \quad Q=\frac{q}{\sqrt{2}}\alpha_3^{\frac{D-3}{4}} \quad m=\frac{16\pi M}{(D-2)\Sigma_{D-2}^{\kappa}\alpha_3^{\frac{D-3}{4}}}$$

$$4\pi p=\alpha_0(D-1)(D-2)\sqrt{\alpha_3} \qquad \alpha=\alpha_3/\sqrt{\alpha_3}$$

Equation of State

$\alpha_k = 0 \quad k > 3 \quad \beta_k = 0 \quad k > 2$

$$p = \frac{t}{v} - \frac{\kappa(D-3)(D-2)}{4\pi v^2} + \frac{2\alpha\kappa t}{v^3} - \frac{\alpha(D-2)(D-5)}{4\pi v^4} + \frac{3t}{v^5} - \frac{\kappa(D-7)(D-2)}{4\pi v^6} + \frac{q^2}{v^{2(D-2)}} - \frac{h}{v^D}$$

Criticality $\frac{\partial p}{\partial v} = \frac{\partial^2 p}{\partial v^2} = 0$

$$h = \frac{4(2D-5)(D-2)^2 v_c^{D-6}}{\pi D(D-4)} \qquad v_c = 15^{1/4}$$

$$q^2 = \frac{2(D-1)(D-2)v_c^{2D-10}}{\pi(D-4)} \qquad \alpha = \sqrt{5/3}$$

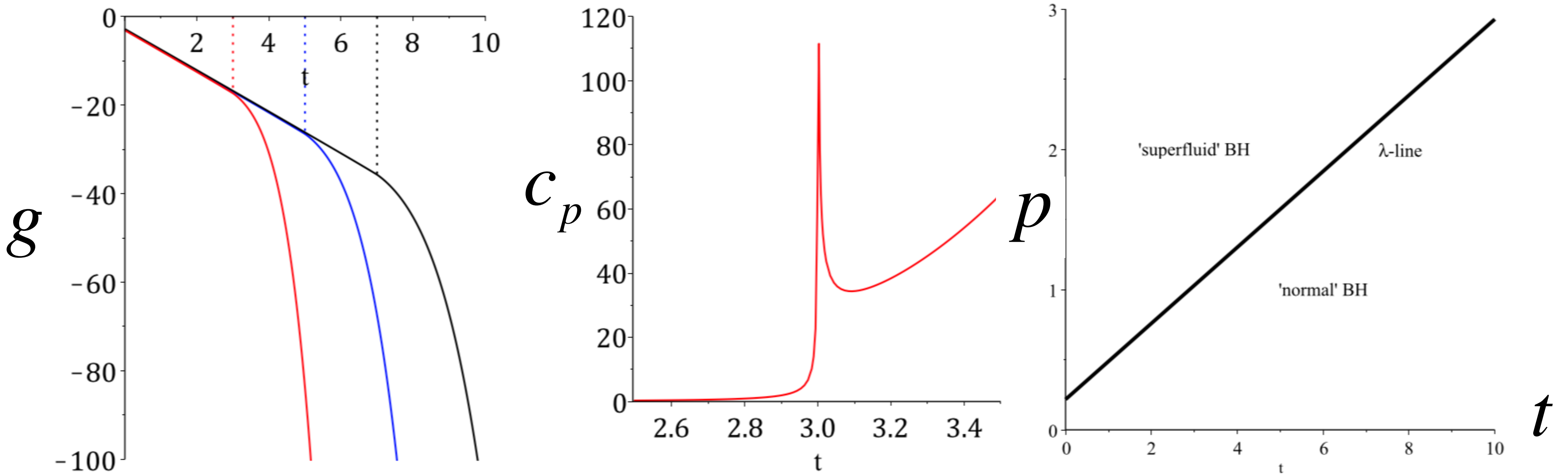
$$p_c = \left[\frac{8(15)^{3/4}}{225}\right] t_c + \frac{\sqrt{15}(11D-40)(D-1)(D-2)}{900\pi D}$$

for all temperatures t_c

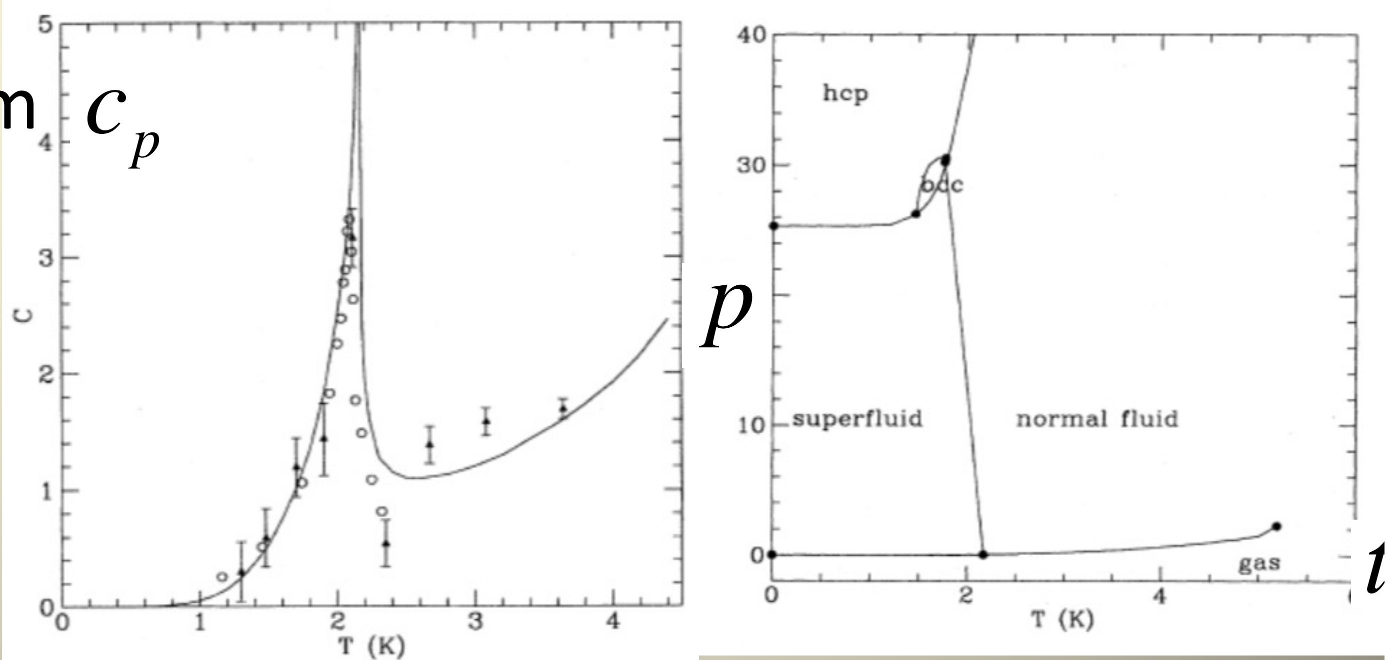
Infinitely many critical points!

Superfluid Black Hole

$$\alpha = 0, \quad \beta = \frac{1}{2} \quad \gamma = 1, \quad \delta = 3$$



Superfluid Helium

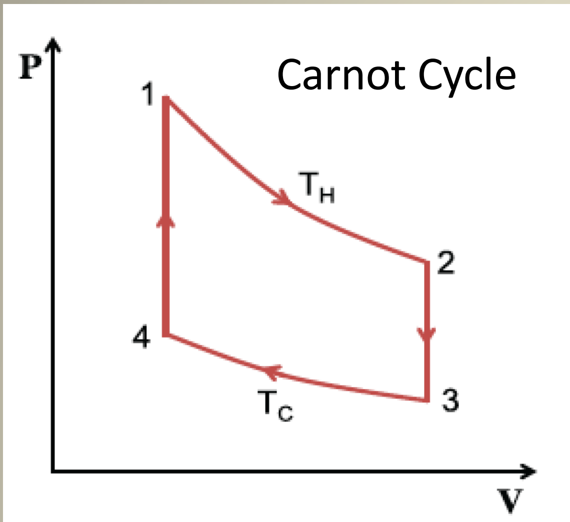


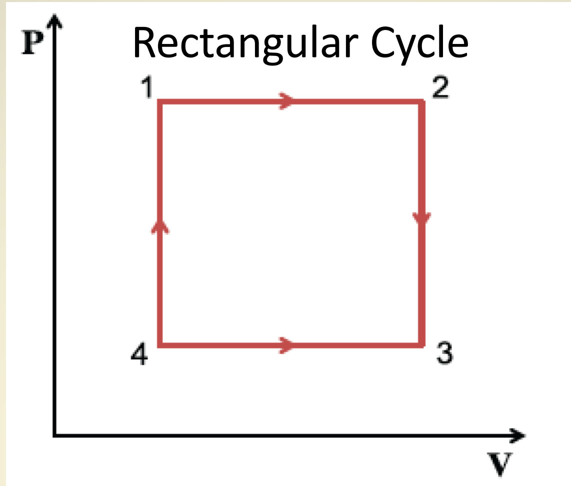
Also seen in cubic quasi-topological gravity!

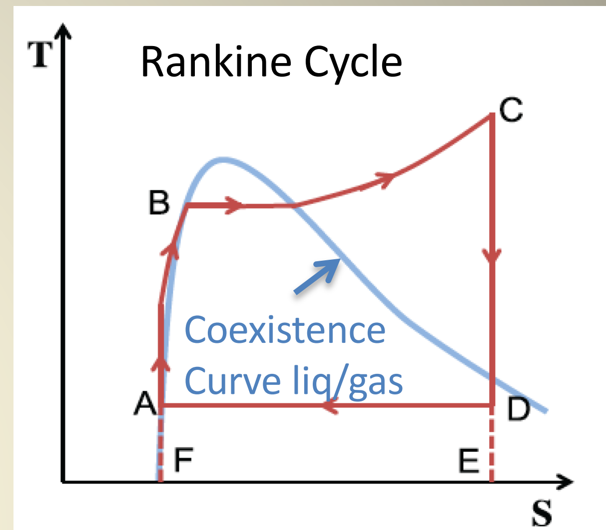
Dykaar/Hennigar/Mann
JHEP 1705 (2017) 045

Black Hole Heat Engines

Johnson CQG **31** (2014) 205002







$$\eta_{\text{Carnot}} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

Black holes as Stirling Engines

$$\eta_{\text{Rec}} = 1 - \frac{M_3 - M_4}{M_2 - M_1}$$

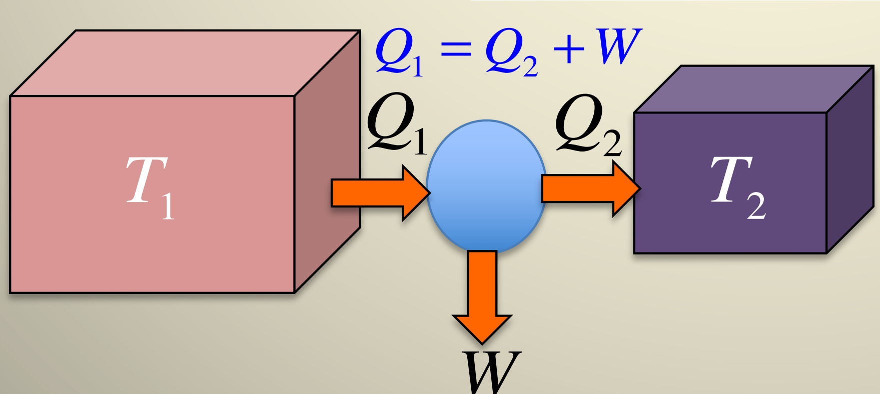
Use to get efficiency of arbitrary cycles

$$\eta_{\text{Rank}} = 1 - \frac{\text{Area}(ADEFA)}{\text{Area}(FABCDEF)}$$

Black holes as steam turbines

Wei/Liu Comm. Theor. Phys. 71 (2019) 711

Heat Engine

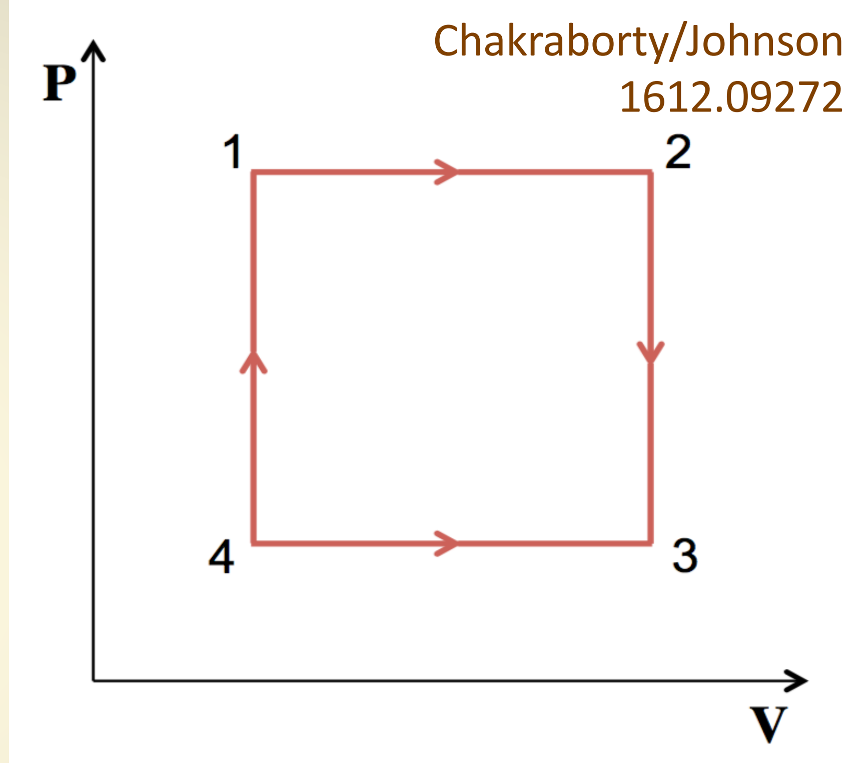


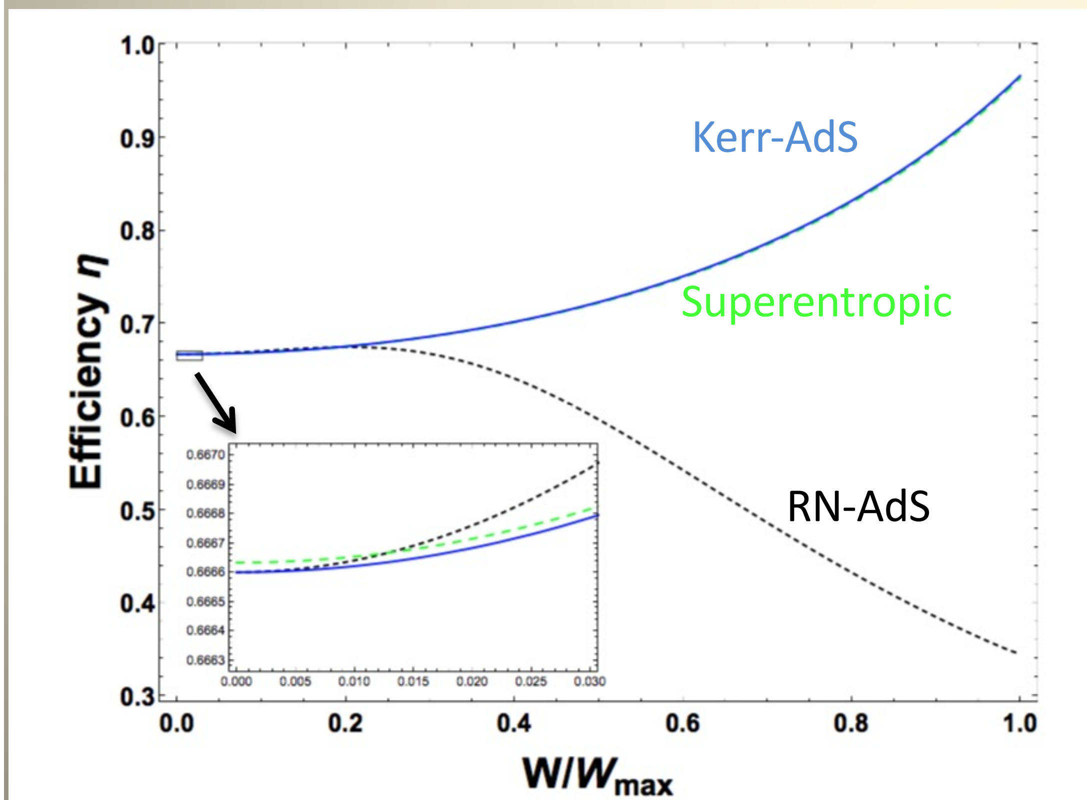
$$\eta_{\text{Carnot}} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

Benchmarking Heat Engines

- Fix an engine cycle (circle)
- Compare efficiency of different black holes

Rectangular Cycle

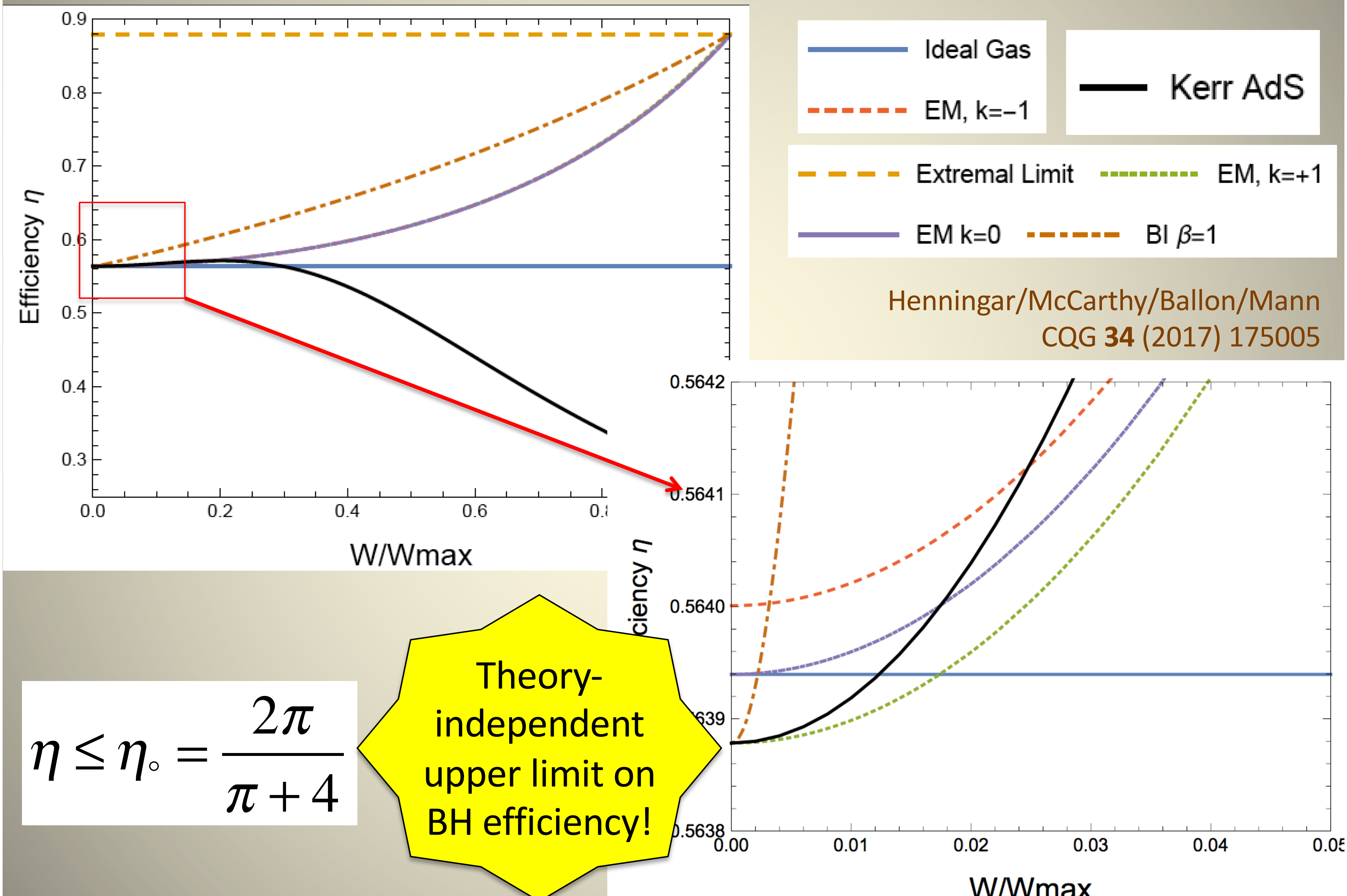




Henningar/McCarthy/Ballon/Mann
CQG **34** (2017) 175005

$$\frac{W}{W_{\max}} = \frac{W(Q,J)}{W(Q_{ex},J_{ex})}$$

Maximizing Efficiency



Henningar/McCarthy/Ballon/Mann
CQG **34** (2017) 175005

$$\eta \leq \eta_\circ = \frac{2\pi}{\pi + 4}$$

Theory-independent upper limit on BH efficiency!

De Sitter Black Hole Chemistry: Cosmic Tension

Dolan/Kastor/Kubiznak/Mann/Traschen
PRD 87 (2013) 104017

$$P = -\frac{\Lambda}{8\pi} = -\frac{3}{8\pi}\frac{1}{\ell^2}$$

Negative Pressure (tension) in de Sitter Spacetime?

Black Holes

$$V_h = \int_\infty dSr_c u_d(\omega^{cd} - \omega^{cd}_{dS}) - \int_{BH} dSr_c u_d \omega^{cd}$$

$$\delta M = T_h \delta S_h + \sum_i (\Omega^i_h - \Omega^i_\infty)\delta J^i + V_h \delta P$$ First Law

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega^i_h - \Omega^i_\infty) J^i - \frac{2}{D-2} P V_h$$ Smarr Relation

dS Horizon

$$V_h = \int_\infty dSr_c u_d(\omega^{cd} - \omega^{cd}_{dS}) - \int_{\mathrm{dS}} dSr_c u_d \omega^{cd}$$

$$\delta M = -T_c \delta S_c + \sum_i (\Omega^i_c - \Omega^i_\infty)\delta J^i + V_c \delta P$$ First Law

$$\frac{D-3}{D-2} M = -T_c S_h + \sum_i (\Omega^i_c - \Omega^i_\infty) J^i - \frac{2}{D-2} P V_c$$ Smarr Relation

Thermodynamics of Kerr-de Sitter Black Holes

- Multiply-rotating Kerr de Sitter Black hole in D dimensions
- 2 horizons at different temperatures

$$ds^2 = -W\left(1-\frac{r^2}{\ell^2}\right)dt^2 + \frac{2m}{U}\left(W dt - \sum_{i=1}^{N}\frac{a_i\mu_i^2 d\varphi_i}{\Xi_i}\right)^2 + \sum_{i=1}^{N}\frac{r^2+a_i^2}{\Xi_i}\left(\mu_i^2 d\varphi_i^2 + d\mu_i^2\right)$$

$$+\frac{U dr^2}{X-2m} + \epsilon r^2 d\nu^2 + \frac{1}{W(\ell^2-r^2)}\left(\sum_{i=1}^{N}\frac{r^2+a_i^2}{\Xi_i}\mu_i d\mu_i + \epsilon r^2 \nu d\nu\right)^2$$

$$W = \sum_{i=1}^{N}\frac{\mu_i^2}{\Xi_i} + \epsilon\nu^2 \qquad X = r^{\epsilon-2}\left(1-\frac{r^2}{\ell^2}\right)\prod_{i=1}^{N}(r^2+a_i^2) \qquad U = \frac{Z\ell^2}{\ell^2-r^2}\left(1-\sum_{i=1}^{N}\frac{a_i^2\mu_i^2}{r^2+a_i^2}\right)$$

$$2\Lambda = \frac{(D-1)(D-2)}{\ell^2} \qquad \Xi_i = 1+\frac{a_i^2}{\ell^2} \qquad \sum_{i=1}^{N}\mu_i^2 + \epsilon\nu^2 = 1$$

$$\epsilon = \begin{cases} 1 & \text{D=even} \\ 0 & \text{D=odd} \end{cases}$$

Even Dim'l Kerr-dS Black Holes

$$M = \frac{m\omega_{D-2}}{4\pi\prod_j \Xi_j}\sum_i \frac{1}{\Xi_i}, \quad J_i = \frac{m a_i \omega_{D-2}}{4\pi \Xi_i \prod_j \Xi_j}$$

$$2m = \frac{1}{\ell^2 r_c}(\ell^2 - r_c^2)\prod_i (r_c^2 + a_i^2) = \frac{1}{\ell^2 r_h}(\ell^2 - r_h^2)\prod_i (r_h^2 + a_i^2)$$

Cosmological Horizon

$$S_c = \frac{\omega_{D-2}}{4}\prod_i \frac{r_c^2 + a_i^2}{\Xi_i} = \frac{A_c}{4}$$

$$T_c = -\frac{r_c}{2\pi\ell^2}\sum_i \frac{(\ell^2 - r_c^2)}{r_c^2 + a_i^2} + \frac{\ell^2 + r_c^2}{4\pi r_c \ell^2}$$

$$\Omega_c^i = \frac{(\ell^2 - r_c^2)a_i}{\ell^2\left(r_c^2 + a_i^2\right)}$$

$$V_c = \frac{r_c A_c}{D-1} + \frac{8\pi}{(D-1)(D-2)}\sum_i a_i J_i \,.$$

Black Hole Horizon

$$S_h = \frac{\omega_{D-2}}{4}\prod_i \frac{r_h^2 + a_i^2}{\Xi_i} = \frac{A_h}{4}$$

$$T_h = \frac{r_h}{2\pi\ell^2}\sum_i \frac{(\ell^2 - r_h^2)}{r_h^2 + a_i^2} - \frac{\ell^2 + r_h^2}{4\pi r_h \ell^2}$$

$$\Omega_h^i = \frac{(\ell^2 - r_h^2)a_i}{\ell^2\left(r_h^2 + a_i^2\right)}$$

$$V_h = \frac{r_h A_h}{D-1} + \frac{8\pi}{(D-1)(D-2)}\sum_i a_i J_i \,.$$

Odd Dim'l Kerr-dS Black Holes

$$M = \frac{m\omega_{D-2}}{4\pi\prod_j \Xi_j}\left(\sum_i \frac{1}{\Xi_i} - \frac{1}{2}\right) \qquad J_i = \frac{m a_i \omega_{D-2}}{4\pi \Xi_i \prod_j \Xi_j}$$

$$2m = \frac{1}{\ell^2 r_c}(\ell^2 - r_c^2)\prod_i (r_c^2 + a_i^2) = \frac{1}{\ell^2 r_h}(\ell^2 - r_h^2)\prod_i (r_h^2 + a_i^2)$$

Cosmological Horizon

$$S_c = \frac{\omega_{D-2}}{4 r_c}\prod_i \frac{r_c^2 + a_i^2}{\Xi_i} = \frac{A_c}{4}$$

$$T_c = -\frac{r_c}{2\pi\ell^2}\sum_i \frac{(\ell^2 - r_c^2)}{r_c^2 + a_i^2} + \frac{1}{2\pi r_c}$$

$$\Omega_c^i = \frac{(\ell^2 - r_c^2) a_i}{\ell^2\left(r_c^2 + a_i^2\right)}$$

$$V_c = \frac{r_c A_c}{D-1} + \frac{8\pi}{(D-1)(D-2)}\sum_i a_i J_i \,.$$

Black Hole Horizon

$$S_h = \frac{\omega_{D-2}}{4 r_h}\prod_i \frac{r_h^2 + a_i^2}{\Xi_i} = \frac{A_h}{4}$$

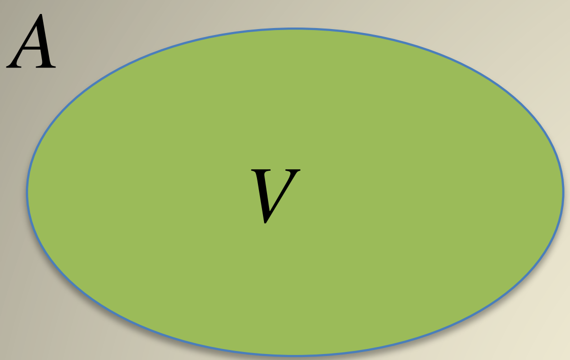
$$T_h = \frac{r_h}{2\pi\ell^2}\sum_i \frac{(\ell^2 - r_h^2)}{r_h^2 + a_i^2} - \frac{1}{2\pi r_h}$$

$$\Omega_h^i = \frac{(\ell^2 - r_h^2) a_i}{\ell^2\left(r_h^2 + a_i^2\right)}$$

$$V_h = \frac{r_h A_h}{D-1} + \frac{8\pi}{(D-1)(D-2)}\sum_i a_i J_i \,.$$

(Reverse) Isoperimetric Inequality?

Cvetic/Gibbons/Kubiznak/Pope
PRD84 (2011) 024037



$$R = \left(\frac{(D-1)V}{\omega_{D-2}}\right)^{1/D-1}\left(\frac{\omega_{D-2}}{A}\right)^{1/D-2} \qquad R \le 1$$

Kerr-(A)dS Black Hole

$$A_h = \frac{\omega_{D-2}}{r_h^{1-\epsilon}}\prod_i \frac{r_c^2 + a_i^2}{\Xi_i}$$

$$V_h = \frac{r_h A_h}{D-1} + \frac{8\pi}{(D-1)(D-2)}\sum_i a_i J_i = \frac{r_h A_h}{D-1}\left[1 + \frac{\ell^2 \pm r_h^2}{(D-2)\ell^2 r_h^2}\sum_i \frac{a_i^2}{\Xi_i}\right]$$

$$R^{D-1} = r_h\left[1 + \frac{z}{D-2}\right]\left[\frac{1}{r_h^{1-\epsilon}}\prod_i \frac{(r_h^2 + a_i^2)}{\Xi_i}\right]^{\frac{1}{2-D}} = \left[1 + \frac{z}{D-2}\right]\left[\prod_i \frac{(r_h^2 + a_i^2)}{r_h^2 \Xi_i}\right]^{\frac{1}{2-D}}$$

$$\ge \left[1 + \frac{z}{D-2}\right]\left[\frac{2}{D-1}\left(\sum_i \frac{1}{\Xi_i} + \sum_i \frac{a_i^2}{r_h^2 \Xi_i}\right)\right]^{\frac{D-1}{4-2D}} = \left[1 + \frac{z}{D-2}\right]\left[1 + \frac{2z}{D-1}\right]^{\frac{D-1}{4-2D}} \equiv F(z)$$

$$F(0) = 1 \qquad \frac{dF(z)}{dz} > 0 \quad \Rightarrow \quad F(z) \ge 1 \quad \Rightarrow \quad R \ge 1$$

Cosmic Volume

Mbarek/Mann
PLB 765 (2017) 352

Can we understand cosmic volume without black hole volume?

Yes! With cosmic solitons!

Soliton: a bubble in spacetime!

Clarkson/Mann
PRL 96 (2006) 051104

- Geometry depends on relative size of the soliton and the
- No black hole horizon!
- Can now have a cosmological horizon surrounding soliton
- Obtained a number of results depending on mass/energy of the soliton and its size relative to the cosmic horizon

$$ds^2 = -g(r)dt^2 + \left(\frac{2r}{d}\right)^2 f(r)\left[d\psi + \sum_{i=1}^{k}\cos(\theta_i)d\phi_i\right]^2_{\ell^2} + \frac{dr^2}{g(r)f(r)} + \frac{r^2}{d}\sum_{i=1}^{k} d\Sigma^2_{2(i)}$$

compact space

$$f(r) = 1 - \frac{a^d}{r^d} \qquad g(r) = 1 - \frac{r^2}{\ell^2} \qquad \Lambda = +\frac{(D-2)(D-2)}{2\ell^2}$$

$$d = 2k+2 = D-1$$

$$d\Sigma^2_{2(i)} = d\theta_i^2 + \sin^2(\theta_i)d\phi_i^2$$

Regularity

$$\Delta\psi = \frac{4\pi}{p} = \frac{8\pi}{\left|r^2 f'(r)\right|_{r=a}} = 2\pi$$

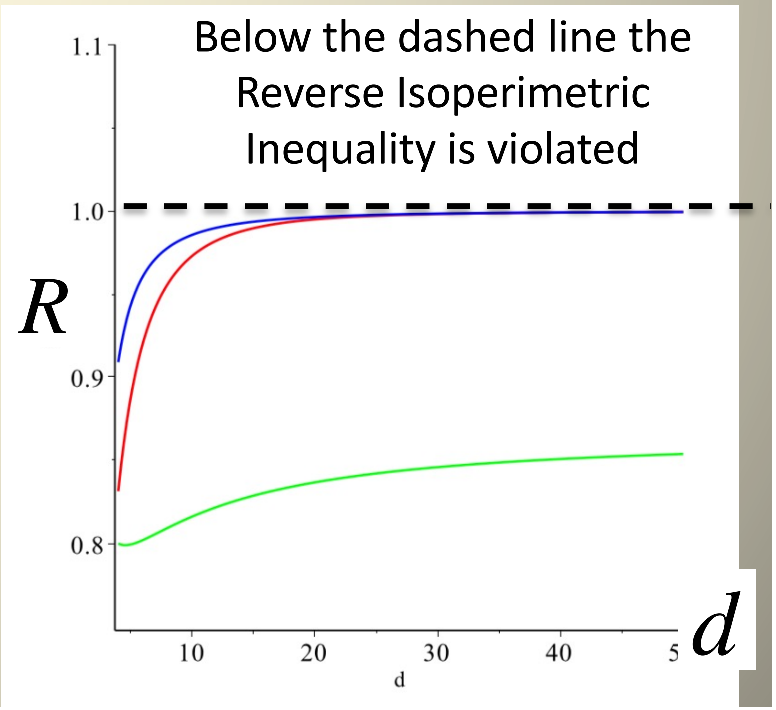
Clarkson/Mann
PRL 96 (2006) 051104

- Geometry depends on relative size of a/ℓ
- No black hole horizon! $r \geq a$
- Can now have a cosmological horizon surrounding soliton

$$M_{in/out} = \pm\frac{Ka^d}{4\pi\ell^2} + m_{\frac{d}{2}}\ell^{d-2}$$

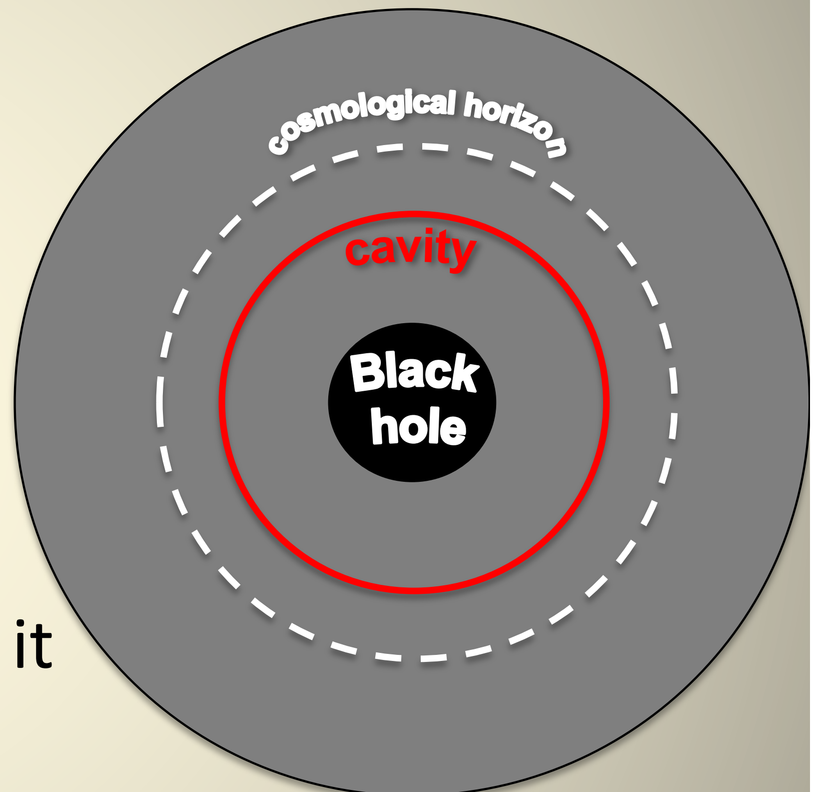
$$V_{in/out} = -\frac{2K}{d-1}a^d + \left(\pm\frac{8(d-2)}{d(d-1)}\pi m_{\frac{d}{2}} + \frac{4K}{d}\right)\ell^d$$

Mbarek/Mann
PLB 765 (2017) 352



De Sitter Black Hole Chemistry

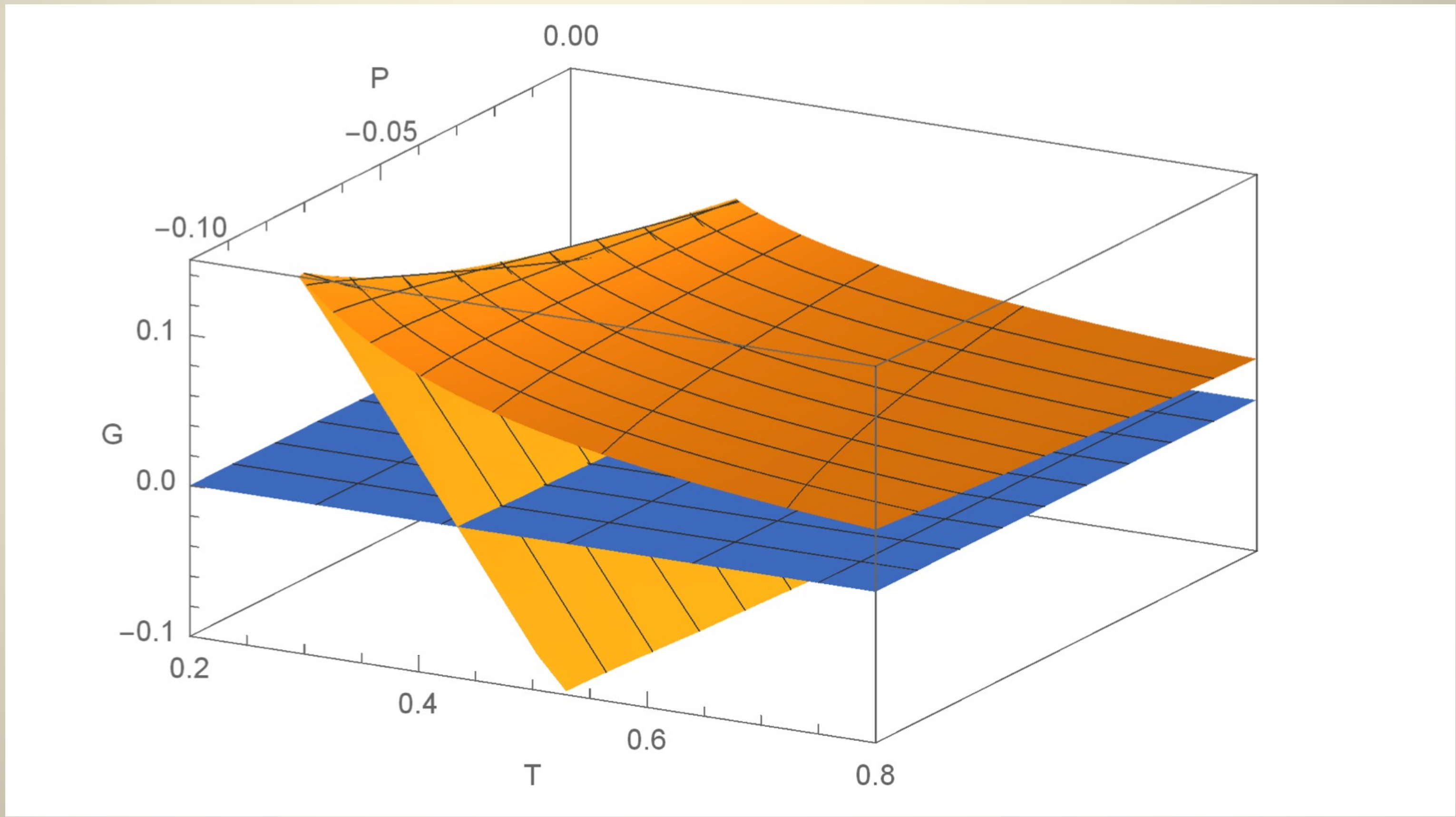
- Black Holes in de Sitter space
  - Two horizons: black hole and cosmological
  - Two temperatures
    → no thermodynamic equilibrium
- Solution?
  - Place black hole in a cavity
  - Control cavity temperature so that it has the same redshifted value as expected from the black hole
- Positive Cosmological Constant → negative P
  - system under tension



SdS Black Holes in a Cavity

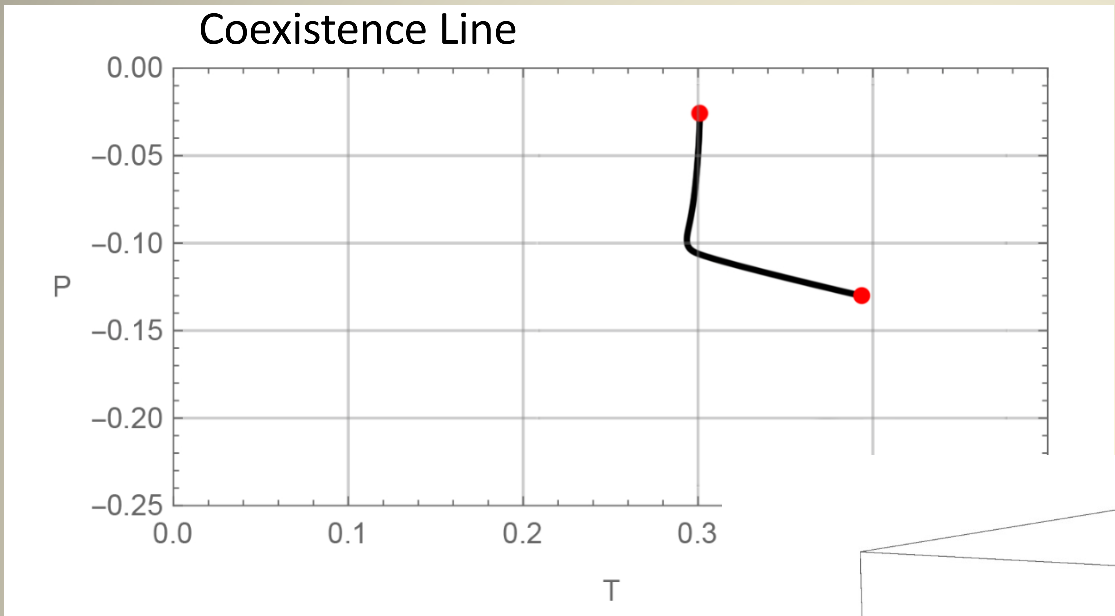
F. Simovic and R.B. Mann
CQG 36 (2019) 014002

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad f(r) = 1 - \frac{r^2}{\ell^2} - \frac{2M}{r}$$



Tension-dependent line of
Hawking-Page transitions

Charged dS Black Holes in a Cavity

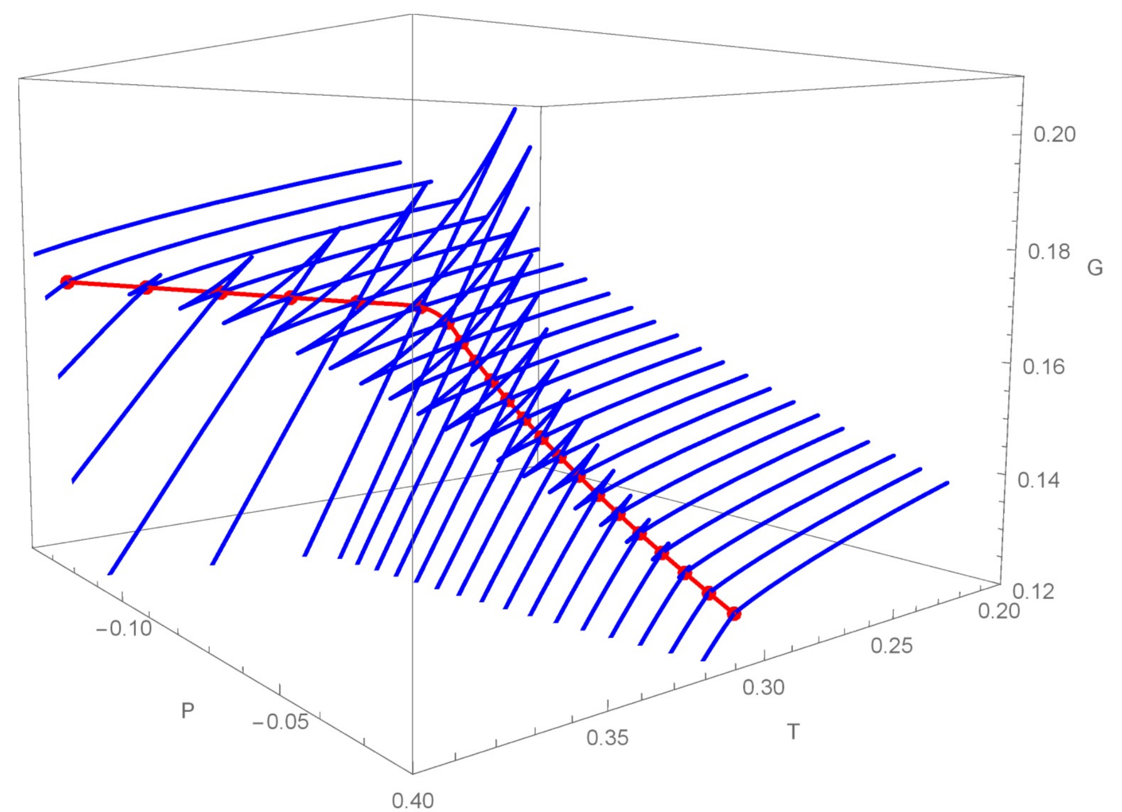


$$f(r) = 1 - \frac{r^2}{\ell^2} - \frac{2M}{r} + \frac{Q^2}{r^2}$$

F. Simovic and R.B. Mann
CQG 36 (2019) 014002
F. Simovic and R.B. Mann
JHEP 1905 (2019) 136

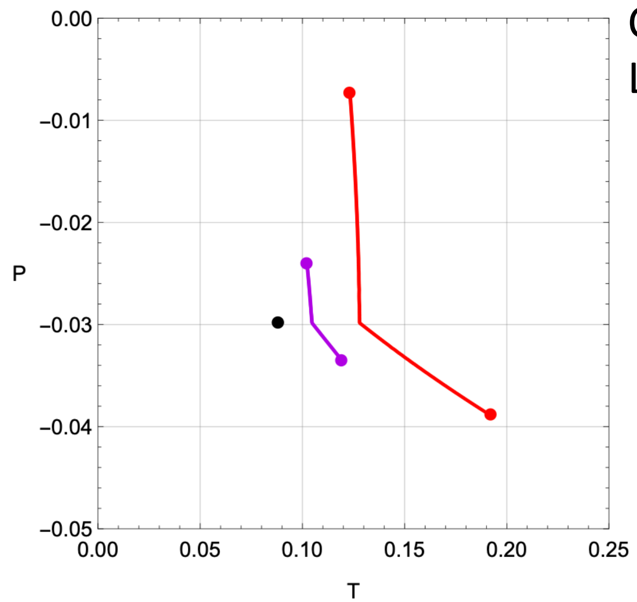
Also can get re-entrant phase transitions in the Born-Infeld case!

Swallowtube

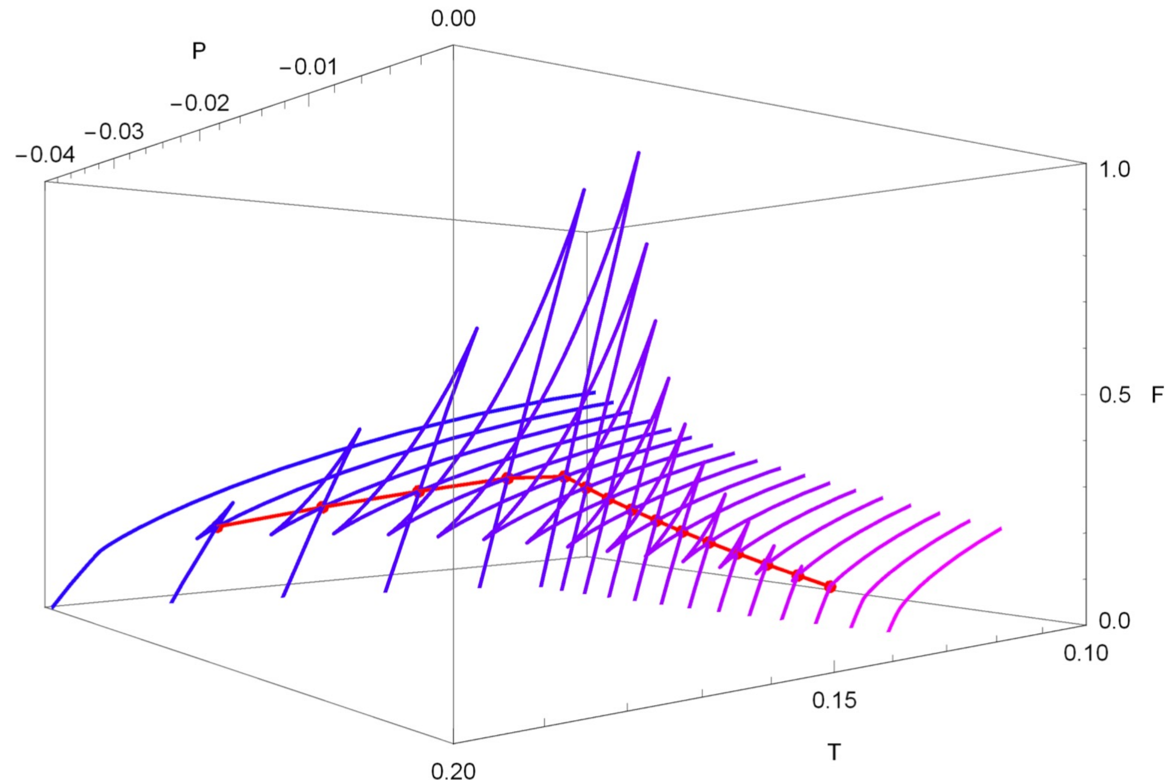


Gauss-Bonnet Swallowtubes

S. Haroon/R. Hennigar/F. Simovic/R.B. Mann
PRD 101 (2020) 08541



Coexistence Lines



Similar Phenomena in 4DEGB and Conformally Coupled Einstein-Maxwell-Dilaton

G. Marks/F. Simovic/R.B. Mann
PRD 104 (2021) 104056

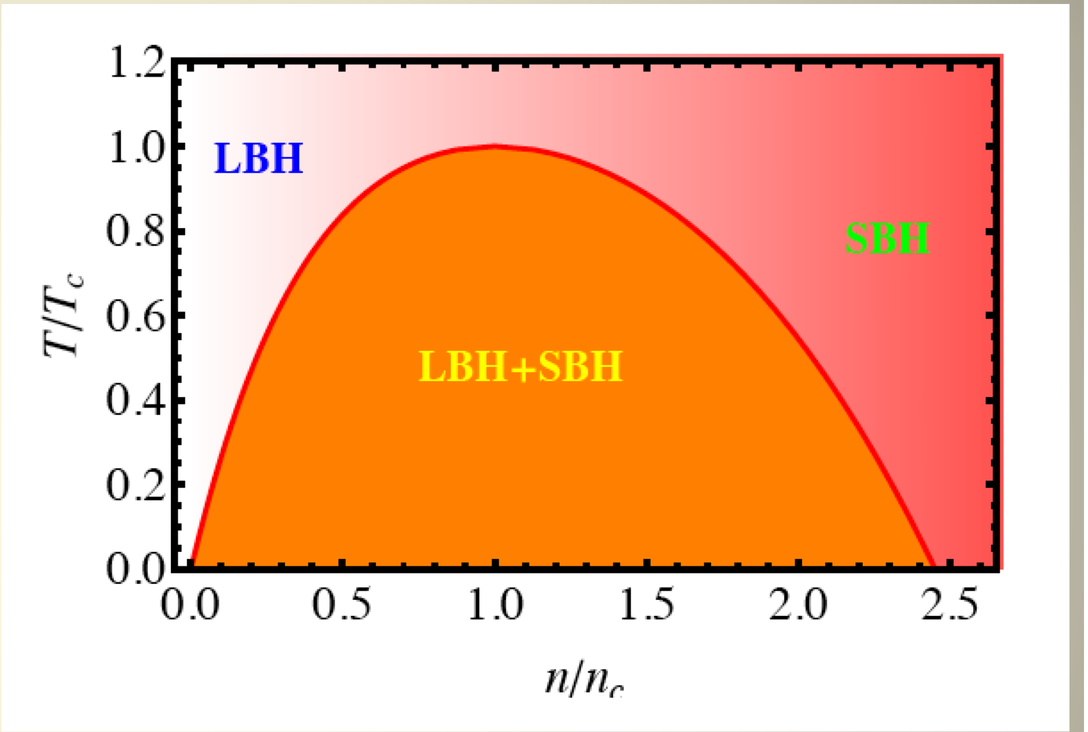
D. Fusco/F. Simovic/R.B. Mann
JHEP 02 (2021) 219

Black Hole Molecules?

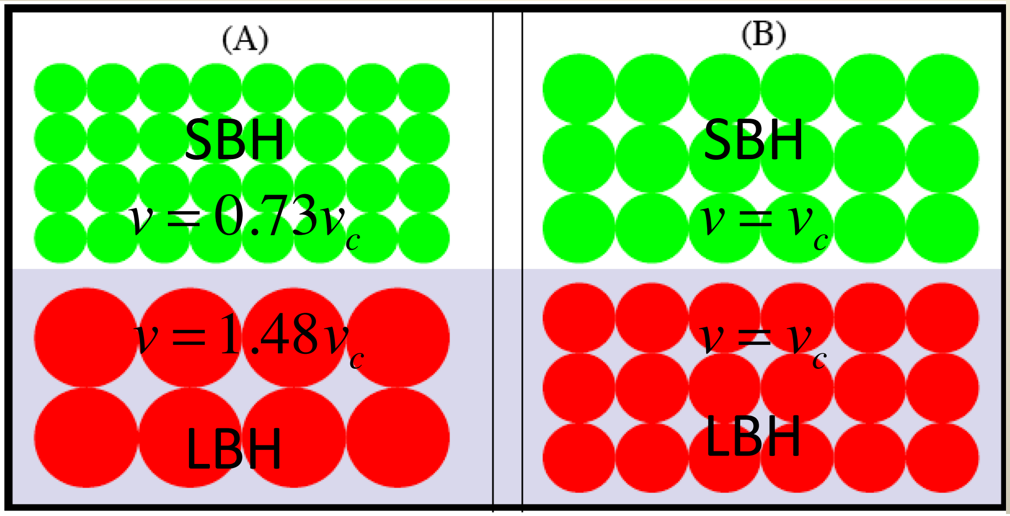
S.W. Wei Y. Liu
PRL **115** (2015) 11132

- "If you can heat it, it has microscopic structure"
  - (L. Boltzmann)

- Perhaps the black hole degrees of freedom are molecular?



$P = 0.85P_c$
$T = 0.94T_c$



$P = P_c$
$T = T_c$

Repulsive Black Hole Microstructure?

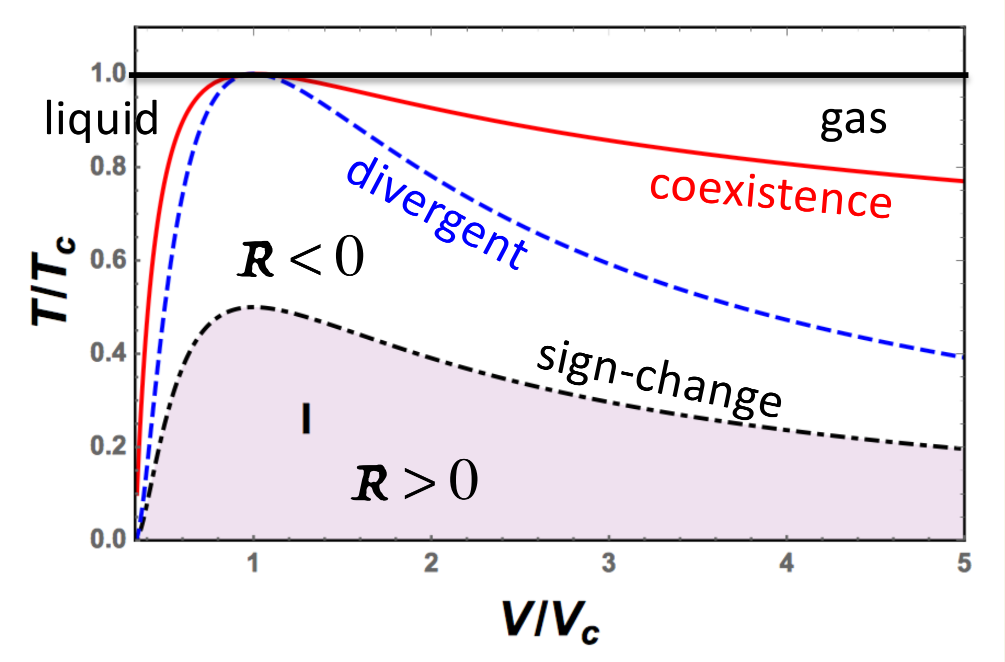
S.W. Wei Y. Liu R.B. Mann
PRL **123** (2019) 071103

Ruppeiner Thermodynamic Curvature

$R > 0 \Rightarrow$ Repulsive

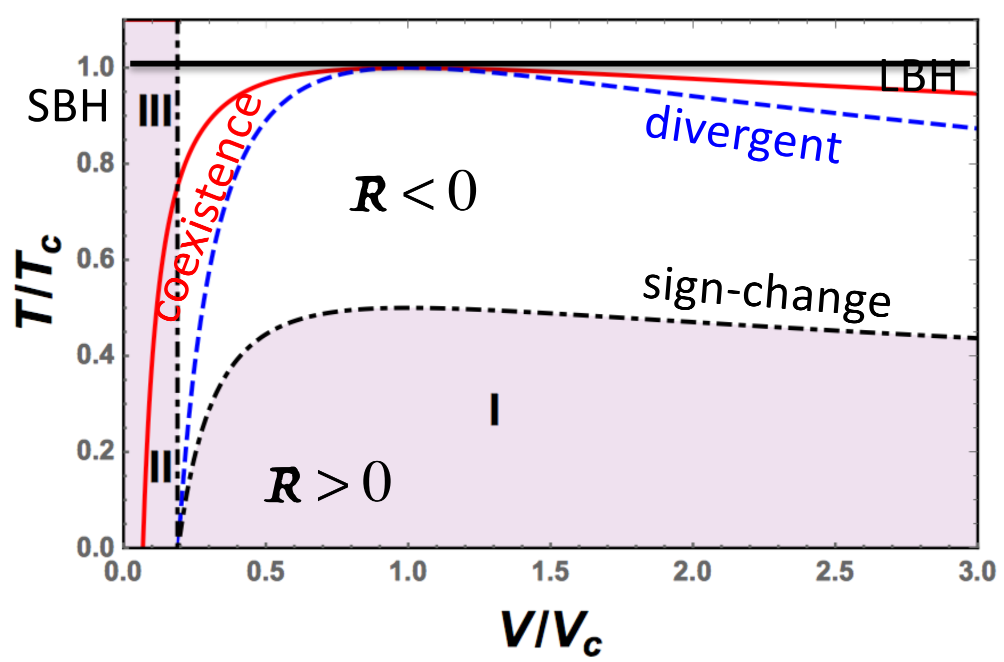
$R < 0 \Rightarrow$ Attractive

$$R = -\frac{1}{8}\left|1-\frac{T}{T_c}\right|^{-2} + \cdots$$



Van der Waals

Attractive microstructure everywhere above coexistence line



Charged AdS Black Hole

LBH: Attractive microstructure

SBH: Repulsive microstructure everywhere above coexistence line